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New Constraints on the Time-variation of the Dark Energy Equation of State from Current Supernova Data

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Understanding the origin of the recently discovered accelerated expansion of the Universe poses one of the greatest challenges in physics today. Due to a lack of a fundamental theory to test, major observational efforts are targeted at characterizing the underlying cause. If the cause is a dark energy, the equation of state w and its possible time evolution will hold clues about its origin. In order to exploit the information from ongoing and upcoming observations it is extremely important to develop a robust and accurate reconstruction approach with controlled errors for the dark energy equation of state. We introduce a new, nonparametric, direct reconstruction method based on Gaussian process modeling. We apply this method to recent supernova measurements and for the first time reliably reconstruct the continuous history of w out to redshift $z = 1.5$.

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Only a decade has passed since the astonishing discovery of a mysterious agent driving an accelerated expansion of the Universe [1, 2]. Since confirmed by different probes, this discovery has been hailed as the harbinger of a revolution in fundamental physics and cosmology. Why is there so much excitement? Simply because no fundamental understanding exists: Cosmic acceleration demands completely new physics. It challenges basic notions of quantum theory, general relativity, and the fundamental make-up of matter. Currently, the two most popular explanations are a dark energy, maybe driven by a scalar field, or a modification of general relativity on the largest scales.

In the absence of any compelling theory to explain the observations, the main target of current and future cosmological missions is to first characterize the underlying cause for the accelerated expansion. In the case of dark energy, constraints on the equation of state $w = p/\rho$ and its possible time dependence are the major targets for future surveys. Any deviation from $w = const.$ would provide clues about the origin of the accelerated expansion. Currently, observations are consistent with a cosmological constant, $w = -1$, at the 10% level [3] without hints on a possible time variation. In order to extract such information from cosmological data, a reliable and robust reconstruction method for $w(z)$ is crucial, since "direct reconstruction is the only approach that is truly model independent" [4]. In this paper, we introduce a new nonparametric method for reconstructing $w(z)$ based on Gaussian Process (GP) modeling.

For supernova data, the reconstruction task can be summarized as follows. The data is given in form of the distance modulus $\mu(z)$ defined as:

$$\mu(z) = m_B - M = 5 \log_{10} D_L(z) + 25. \qquad (1)$$

The luminosity distance $D_L(z)$ is connected to the Hubble expansion rate and therefore to the dark energy equation of state $w(z)$ via:

$$\begin{aligned} D_L(z) &= \frac{c(1+z)}{H_0} \int_0^z \frac{ds}{h(s)} \qquad (2) \\ &= \frac{c(1+z)}{H_0} \int_0^z ds \left[\Omega_m (1+s)^3 \right. \\ &\quad \left. + (1-\Omega_m)(1+s)^3 \exp\left(3 \int_0^s \frac{w(u)}{1+u} du\right)\right]^{-\frac{1}{2}}, \end{aligned}$$

where $h(z) = H(z)/H_0$. We assume here spatial flatness which is well justified from combined cosmic microwave background and baryon acoustic oscillation measurements [5]. Equation (2) defines our reconstruction task for $w(z)$, a classic statistical inverse problem, where we have to solve for a function $w(z)$ and two parameters (H_0 and Ω_m) given noisy data.

To make the problem tractable, there are different ways to proceed. (i) Assume a parametrized form for $w(z)$ and estimate the associated parameters. This approach is currently most commonly used and the parametric forms either assume $w = const.$ or allow for a small time variation such as $w = w_0 - w_1 z/(1 + z)$, where w_0 and w_1 are constant [6]. (ii) Pick a simple local basis representation for $w(z)$ (bins, wavelets), and estimate the associated coefficients (effectively a piecewise constant description), use Principal Component Analysis (PCA) if needed to work with eigenmodes defined as linear combinations of bins [7]. (iii) Follow a procedure similar to (ii) – without PCA – but actually use (filtered) numerical derivatives to estimate w(z) [8]. (iv) Assume a general functional representation for $w(z)$ and estimate the properties thereof (we will follow this approach in the current paper). Methods (i), (ii), and (iv) can all be carried out using a Bayesian approach and explore posteriors by Markov Chain Monte Carlo (MCMC) methods, whereas (iii) – as carried out in the literature – represents a different class of approach to the inverse problem. It requires taking numerical derivatives which is generally a difficult task and an error theory for this method seems hard to develop. It will never be as good as the other methods – if it works, the others should already work better.

Approach (i) would not fare well if $w(z)$ has indeed a non-trivial time dependence. The number of parameters is too small to capture a time dependence reliably and the specific functional form assumed can easily bias the result for the temporal behavior of w. In principle, one could use more parameters (e.g. in a Chebyshev polynomial expansion or a harmonic oscillator basis), and include as many as the data requires. In practice, the current data quality does not hold enough information to go beyond a one- or two-parameter fit and limits the usefulness of this approach. Methods (ii) and (iv) in effect apply different philosophies – (ii) applies a local view of the reconstruction (z bins), whereas (iv) attempts to sample the posterior continuously in z. In some sense, (ii) is mildly parametric because of the choice of a piecewise continuous representation ($w = const.$ is just the one-bin limit of this). It also forces an unphysical view of $w(z)$ since the actual $w(z)$ is not piecewise constant. In contrast, method (iv) can be performed in a fully nonparametric fashion. As such, it is more flexible and more general than the others.

Our new approach – based on GP modeling – is an example for a realization of method (iv). It enables us to pick up non-trivial time dependencies in $w(z)$ reliably (see Ref. [10] for examples based on simulated data). The basic idea is to use a large family of possible functional forms for $w(z)$ (we choose GP models because of their flexibility) and take advantage of the integral form of Eq. (2). Using a Bayesian approach to explore posterior distributions via MCMC we not only obtain a continuous best-fit realization for $w(z)$ but at the same time optimize the GP model parameters, informed by the actual data.

There seems to be a confusion in the literature that a general nonparametric reconstruction involves taking a second derivative. Formally this is true but not practically – our approach does not involve any derivatives. Instead, we are "inverting" an integral equation, ill-posed because the operator to be treated is a complicated smoothing operator (a double integral). To make the problem well-behaved we make relatively mild smoothness assumptions about $w(z)$ which is justified if the origin of dark energy is described by a well defined theoretical model.

For our analysis we focus on one of the most recently available supernova data sets from Hicken et al. [3]. This data set combines the so-called Union data set [11] with new measurements of low redshift supernovae to form the Constitution set. The data set has been analyzed in Ref. [3], using different light curve fitters for fitting the supernova light curves. They investigate the original version of the Spectral Adaptive Lightcurve Template (SALT) fitter [12], which is based on spectral template

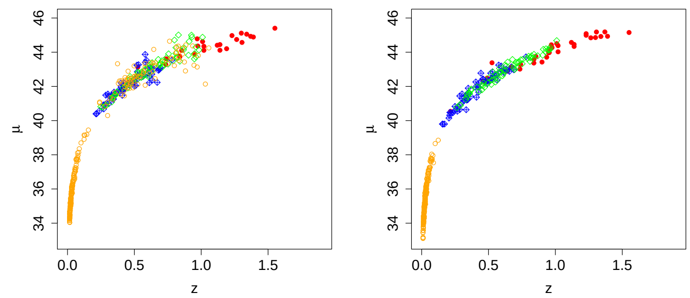

FIG. 1: Data set from Ref. [3], combining the Union data set from Ref. [11] and a new set of low redshift supernovae. The left panel shows the distance modulus μ versus redshift z obtained using the SALT fitter, the right panel shows μ versus z obtained using MLCS17. The colors indicate different subsets of data. It is interesting to note that the data points from MLCS17 are much tighter around a mean than the results from the SALT fitter.

fitting and a newer version, SALT2 [13] based on principle component analysis (PCA). In addition, they analyze the data using the Multicolor Light Curve Shape 2k2 (MLCS2k2) [14] fitter with two different parameter settings for the extinction ratio R_V. The results obtained from these methods are consistent within error bars and in agreement with a cosmological constant.

We carry out an analysis of the results from the SALT fitter and MLCS2k2 with $R_V = 1.7$, denoted as MLCS17 in Ref. [3]. The data are given in Tables 1 and 4 in Ref. [3] including an estimate for the error for the distance modulus μ (the tables contain what is referred to in Ref. [3] as "minimal cut"). Figure 1 shows the two resulting data sets for μ as a function of redshift z. The SALT data set shown in the left panel contains 397 data points, while the MLCS17 data set contains 372 data points.

A GP model assumes that $w(z_1), ..., w(z_n)$, for any collection of $z_1, ..., z_n$, follow a multivariate Gaussian distribution. Here we use a mean of negative one and power family covariance function written as (the constant ρ should not be confused with the density)

$$K(z, z') = \kappa^2 \rho^{|z-z'|^\alpha}. \tag{3}$$

The value of α influences the smoothness of the GP realizations: for $\alpha = 2$, the realizations are smooth with infinitely many derivatives, while $\alpha = 1$ leads to rougher realizations suited to modeling continuous non-differentiable functions. We use both values for α in our analysis, the results are very similar. The mean of the GP is taken to be fixed to improve the stability of the MCMC (we explored other means and found very similar results that all had posteriors that tended toward negative one). ρ has a prior of $Beta(6, 1)$ and κ^2 has a vague prior $IG(25, 9)$. Ω_m and m_B are given priors based on currently available estimates. We marginalize over these two parameters to obtain the final results.

Following the notation of Eqn. (2) we set up the following GP for w:

$$w(u) \sim \mathrm{GP}(-1, K(u, u')). \tag{4}$$

Recall that we have to integrate over $w(u)$ (Eqn. 2):

$$y(s) = \int_0^s \frac{w(u)}{1+u} du. \tag{5}$$

We use the fact that the integral of a GP is also a GP with mean and correlation dependent on the original GP [15]. The integral of a GP can be found by integrating the correlation function. We therefore set up a second GP for $y(s)$:

$$y(s) \sim \mathrm{GP}\left(-\ln(1+s), \kappa^2 \int_0^s \int_0^{s'} \frac{\rho^{u-u'} du du'}{(1+u)(1+u')}\right). \tag{6}$$

The mean value for this GP is simply obtained by solving the integral in Eqn. (5) for the mean value of the GP for $w(u)$, negative one. We can now construct a joint GP for $y(s)$ and $w(u)$:

$$\begin{bmatrix} y(s) \\ w(u) \end{bmatrix} \sim \mathrm{MVN}\left[\begin{bmatrix} -\ln(1+s) \\ -1 \end{bmatrix} \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}\right], \tag{7}$$

with

$$\Sigma_{11} = \kappa^2 \int_0^s \int_0^{s'} \frac{\rho^{u-u'} du du'}{(1+u)(1+u')}, \tag{8}$$

$$\Sigma_{22} = \kappa^2 \rho^{|z-z'|^\alpha}, \tag{9}$$

$$\Sigma_{12} = \kappa^2 \int_0^s \frac{\rho^{u-u'} du}{(1+u)}. \tag{10}$$

The mean for $y(s)$ given $w(u)$ can be found through the following relation:

$$y(s)|w(u) = -\ln(1+s) + \Sigma_{12}\Sigma_{22}^{-1}\left(w(u) - (-1)\right). \tag{11}$$

Now only the outer integral is left to be solved for in Eqn. (2), and this can be computed by standard numerical methods. Note that we never have to calculate the double integral in Σ_{11}, which would be numerically expensive, because it does not appear in our relationship in Eqn. (11). In addition, the method does not require the inversion of one large covariance matrix that would typically be needed to have enough partitions to do the outer integration and is therefore efficient. More details about each step in the GP model algorithm are given in Ref. [10].

Our final results for the reconstructed dark energy equation of state $w(z)$ are shown in Figures 2 and 3. Figure 2 shows the results we obtain from a GP model with a Gaussian correlation function ($\alpha \simeq 2$) while the results in Figure 3 are based on an exponential correlation function ($\alpha = 1$). The results are very similar, the Gaussian correlation function leads to a slightly smoother prediction. For the data set based on the SALT fitter, $w(z)$ is

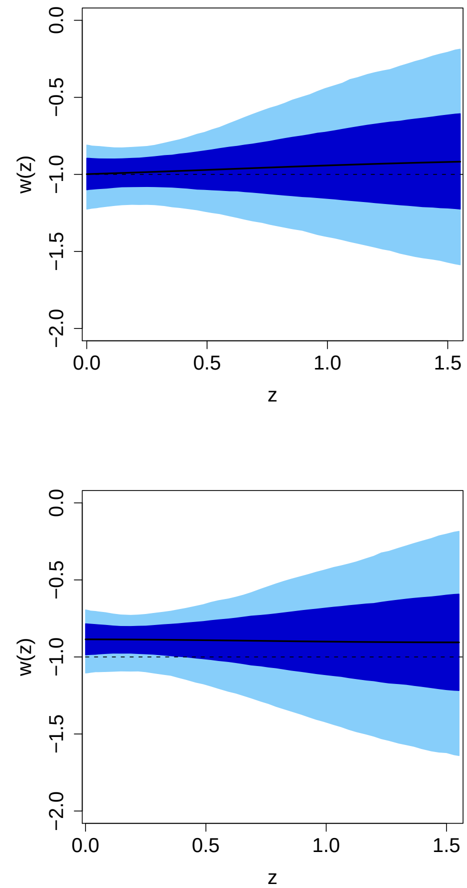

FIG. 2: Nonparametric reconstruction of $w(z)$ based on GP modeling from the data shown in Figure 1. The upper panel displays the result for the data obtained using the SALT fitter, the lower panel shows results based on data extraction using MLCS17. Both results are in agreement with a cosmological constant (black dashed line). The dark blue shaded region indicates the 68% confidence level, while the light blue region extends to 95%.

very close to -1 at redshifts close to zero and rises slightly to $w = -0.9$ at redshift $z = 1.5$. For the MLCS17 based data set, $w(z)$ is slightly above -1 over the whole redshift range at approximately $w = -0.9$. The results are consistent with each other and with a cosmological constant $w = -1$ within error bars.

In Ref. [3] a combined analysis of supernova data and baryon acoustic oscillation measurements is carried out. Under the assumption $w = const.$ they find for the SALT based data set $w = -0.987^{+0.066}_{-0.068}$ and for the MLCS17 based data set $w = -0.901^{+0.066}_{-0.067}$, consistent with our findings.

To summarize, we have presented a new, nonparametric reconstruction method for the dark energy equation of state based on GP models. Our method allows the acceptance or rejection of classes of dark energy mod-

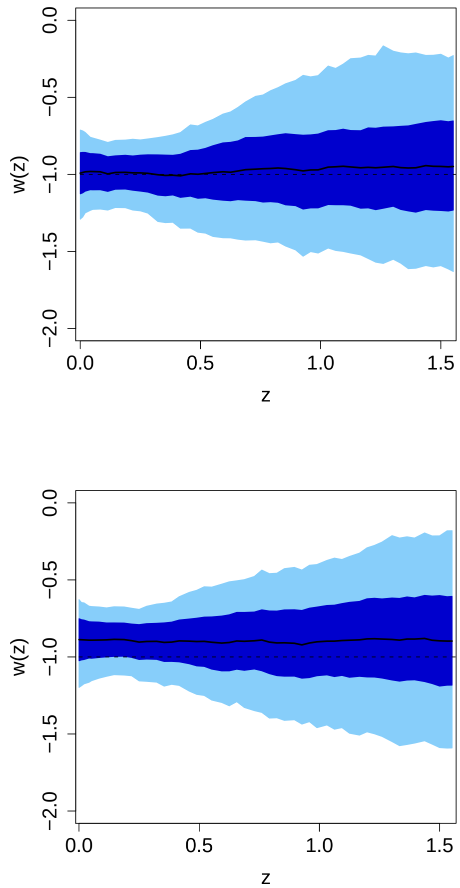

FIG. 3: Same as in Figure 2 for $\alpha = 1$. This choice allows for more variability in reconstructing $w(z)$ leading to a less smooth result. The overall behavior of $w(z)$ is unchanged.

els by providing the probability that $w(z)$ is following a certain trajectory. Our reconstruction approach leads to the most probable behavior of $w(z)$ and has information about how likely a different trajectory is given the current data. Of course, the method relies on some assumptions about $w(z)$ – it should be somewhat smooth and the priors on the GP modeling parameters would not allow for any arbitrary behavior of $w(z)$. But these assumptions are rather mild and can be relaxed when the data quality is improving. We have analyzed recent supernova measurements and reconstructed the redshift dependence of the dark energy equation of state. Our results are consistent with a cosmological constant and with previous findings by other groups. We have carried out careful tests to ensure that our choices of priors and model parameters do not alter the results. As detailed in the paper, this new method has many advantages over previous reconstruction approaches: it does not introduce any artificial biases due to restricted parametric assumptions on w, it does not lose information about the data by smoothing the data itself in order to fit it and obtain $w(z)$ via taking derivatives, and finally it does not introduce arbitrariness to the reconstruction process by choosing a certain number of bins to represent the data or cutting off information by using only a restricted set of basis functions to represent the data.

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