Fast Parameters Extraction of General Three-Dimension Interconnects Using Geometry Independent Measured Equation of Invariance

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ABSTRACT

Measured Equation of Invariance(MEI) is a new concept in computational electromagnetics. It has been demonstrated that the MEI technique can be used to terminate the meshes very close to the object boundary and still strictly preserves the sparsity of the FD equations. Therefore, the final system matrix encountered by MEI is a sparse matrix with size similar to that of integral equation methods. However, complicated Green's function and disagreeable Sommerfeld integrals make the traditional MEI very difficult, if not impossible, to be applied to analyze multilayer and multiconductor interconnects. In this paper, we propose the Geometry Independent MEI(GIMEI) which substantially improved the original MEI method. We use GIMEI for capacitance extraction of general three-dimension VLSI interconnect. Numerical results are in good agreement with published data and those obtained by using FASTCAP from MIT[NW92], while GIMEI are generally an order of magnitude faster than FASTCAP with much less memory usage.

Keywords: interconnects, fast 3D extraction, Measured Equation of Invariance (MEI), geometry independent, measuring loop, capacitance matrix

1 Introduction

Analysis and design of interconnections in high speed VLSI chips, multichip models (MCM's), printed circuit boards (PCB's) and backplanes are gaining importance due to the rapid increase in operating frequencies (with the rise time of digital signals dropping into subnanosecond range) and decrease in feature sizes (with deep submicron process technology). For high speed and high density interconnects, we have to consider the propagation delay and transmission line impedance, together with other effects such as signal degradation caused by transmission line dispersion, signal reflection at discontinuities, crosstalk between adjacent and cross lines, and simultaneous switching noise due to the inductance in power distribution system. And these effects must be quantified in order not to render a fabricated digital circuit inoperable or to distort an analog signal and make it fail to meet specifications. Therefore, it is necessary to develop computationally efficient methods to extract the parasitics of the interconnects.

For an inhomogeneous structure like VLSI interconnects, the modes are hybrid and full-wave approach should be adopted. However, the qausi-static(qausi-TEM) approximations are sufficiently accurate when the transverse components predominates over the longitudinal ones, in other words, the transverse dimensions of the structure are much smaller than wavelength. As a matter of fact, most methods applied to extract the interconnect parameters in recent years used the qausi-TEM approximation, which basically solve the Laplace equation with appropriate boundary conditions. Due to the frequency range of interest for high-speed VLSI is often below twenty gigahertz, we adopt the quasi-TEM assumption. In fact, up to now the static capacitance matrix $[C]$ and inductance matrix $[L]$ of the multilayer and multiconductor interconnect is commonly used in practice for high-speed VLSI, PCB's and MCM's design.

The various procedures to get the solution can be generally classified into two categories. One category is to solve differential Maxwell equations called domain or finite methods, such as Finite Element method(FEM)[Coa87] and Finite Difference method(FDM)[Zem88] [ZT89]. They basically divide the space surrounding the object into meshes, then write local equations at each mesh point, which leads to a sparse matrix system. But the standard FD(or FE) method involves large number of unknowns because they get the solution of the potential distribution over the entire geometry domain and the boundary conditions are usually valid only far from the object. The other category is using the integral equation approach such as Method of Moments[CHMS84] (MoM), the Boundary Element Method[PWG92] (BEM) , and the BEM with multipole acceleration [NW92]. They make meshes on the surface of the object. For multilayer multiconductor interconnects, this means meshes are made either on the surface of each conductor with Green's function for a layered medium which is both mathematically and computationally complex, or on the surfaces of each conductor and all dielectric interfaces but with much simplified Green's function. Compared to FD, this greatly reduces the number of unknowns. But each small piece is either source or field point, and affected by all others, which leads to a full matrix. Therefore, all these methods will either solve a sparse but very large matrix or solve a small but full matrix.

Measured Equation of Invariance(MEI) is a new concept in computational electromagnetics[MPCL92] [HLM94] [HM94] [HML94]. MEI is used to derive the local finite difference (FD) like equation at mesh boundary where the conventional FD approach fails. It is demonstrated that the MEI technique can be used to terminate the meshes very close to the object boundary and still strictly preserves the sparsity of the FD equations. Therefore, the final system matrix encountered by MEI is a sparse matrix with size similar to that of integral equation methods. Therefore, the method of MEI definitely results in dramatic savings in computing time and memory usage compared to other known methods. It has been successfully used to analyze electromagnetic scattering problems, and to analyze microwave integrated circuits. For multilayer and multiconductor structures, however, the deduction of Green's function is very difficult. Also the calculation of the MEI coefficients will encounter many Sommerfeld type integrals. As the result, the calculation of MEI coefficients dominates the total computing time. Therefore, complicated Green's function and disagreeable Sommerfeld integrals make the traditional MEI very difficult, if not impossible, to be applied to analyze multilayer and multiconductor interconnects.

Recently, a MEI variety called Geometry Independent MEI(GIMEI) was proposed[HSD96] which was verified to be extremely computationally efficient, and has been successfully used to solve two-dimension VLSI interconnect problems. Geometry Independent MEI substantially improved the MEI in three key aspects: 1) cancelled the postulate of geometry specific in conventional MEI, 2) avoided the deduction of Green's function in multilayer structure, and 3) avoided the calculation of disagreeable Sommerfeld type integrals. Using this method, the calculation of MEI coefficients only costs a very little part of the total computing time. In this paper, we extended Geometry Independent MEI to compute capacitance matrix of general three-dimension interconnects. The results are in good agreement with published data and those obtained by using FASTCAP from MIT[NW92]. And GIMEI can generally achieve an order of magnitude faster than FASTCAP with significantly less memory usage.

2 Problem Formulation and Generalized FD Equation

A general interconnect configuration is shown in Fig. 1. For an N-conductor system, an $N \times N$ capacitance matrix is defined by

$$Q_i = C_{ii}\Phi_i + \sum_{j=1}^{N} C_{ij}(\Phi_i - \Phi_j) \qquad i = 1, 2, \ldots, N \tag{1}$$

which can be rewritten as:

$$Q_i = \sum_{j=1}^{N} C^s_{ij}\Phi_j \qquad i = 1, 2, \ldots, N \tag{2}$$

where C^s_{ij} is the short circuit capacitance.

We have the transformation:

$$C_{ii} = \sum_{j=1}^{N} C^s_{ij}, \qquad C_{ij} = -C^s_{ij} \qquad for \;\; i \neq j \tag{3}$$

and in this paper, when talking about capacitance, we all refer to the short circuit capacitance.

Now, the parasitic capacitance problem to be considered reduces to the determination of charge on each conductor for known potentials.

We first discretize the geometry in interest into elementary boxes using a three dimensional Cartesian grid shown in Fig.2. The electrical potential can be assumed to be constant inside the elementary boxes and confined at the middle of the box. The mesh points on the metalization can

Figure 1: A general 3D interconnect configuration, the two figures are not related

be treated to be at a constant potential under the qausi-TEM assumption. The boundary of the mesh is treated later when we present the concept of MEI.

The electrical potential function ϕ in the bounded region except those mesh points on conductors of the quasi-static problem satisfies the following Laplace equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{4}$$

Using difference to approximate derivative, we can write the electric potential at each internal mesh point as the linear combination of potentials of neighboring mesh points. Fig.3 shows a general local FD meshes. In the figure, h_x, h_y, and h_z are the discretization distances along x, y, and z directions, the eight subregions of the seven point net may be filled with different media with different relative permittivities. We denote ε_1 be the relative permittivity of the subregion bounded by nodes $0, 2, 4, 5$, ε_2 by nodes $0, 1, 4, 5$, ε_3 by nodes $0, 1, 4, 6$, ε_4 by nodes $0, 2, 4, 6$, and ε_5 to ε_8 corresponds to ε_1 to ε_4 except they rely on the lower part of the Fig.3. By using the loop integral technique[HLM94], the local FD equations for internal node of the mesh can be deduced and written as:

Figure 2: Discretization mesh of the structure

$$\sum_{i=0}^{6} c_i \phi_i = 0 \tag{5}$$

where

$$c_1 = \frac{1}{h_x^2}(\varepsilon_2 + \varepsilon_3 + \varepsilon_6 + \varepsilon_7) \tag{6}$$

$$c_2 = \frac{1}{h_x^2}(\varepsilon_1 + \varepsilon_4 + \varepsilon_5 + \varepsilon_8) \tag{7}$$

$$c_3 = \frac{1}{h_y^2}(\varepsilon_5 + \varepsilon_6 + \varepsilon_7 + \varepsilon_8) \tag{8}$$

Figure 4: A slice cut from Fig.2 illustrating measuring box

where M is the number of nodes that surrounding the node in interest ϕ_0. The node configuration is shown in Fig. 4 which is a slice cut from Fig.2 with the surfaces A and $A\prime$. And the coefficients in Eq.13 are: (i) location dependent, (ii) geometric specific, (iii) invariant to the excitation. Eq. 13 is called measured equation of invariance (MEI), and C_i, $i = 0, \ldots, M$, the coefficients of MEI.

In conventional MEI, the distribution functions, called "metrons", are excited on the conductors and the potential values on the MEI nodes as shown in Fig. 2 and Fig. 4 are obtained from the integrals of the metrons multiplied by Green's function. Substituting the potential values at MEI nodes into MEI (Eq. 13) will lead to a system of linear algebraic equations with respect to the MEI coefficients $C_i(i = 1, \ldots, M)$, where each equation corresponding to one metron. The MEI or MEI coefficients are determined by solving the system of linear algebraic equations. Finally, the potential values at all nodes can be obtained by solving the system of linear algebraic equations which consist of FD equations at interior nodes and MEI at truncated mesh boundary nodes. The coefficient matrix of the system of linear algebraic equations is a sparse matrix since each row contains either seven non-zero elements from FD equations or M(or less) non-zero elements from MEI. Here, M is at most six without considering diagonal nodes. It results in great savings in memory needs compared with BEM or MoM etc. Furthermore, the computing time is proportional to N^2 for solving a sparse matrix equation but N^3 for solving a full matrix equation. The order of coefficient matrix in MEI approach is much less than that in conventional FD methods with absorbing boundary conditions, because MEI can terminate the mesh very close to the region in which we are interested. These properties make the method of MEI a powerful tool for computational electromagnetics.

In addition, solving the dense matrix for each MEI nodes are totally different from solving the dense matrix in integral equation methods, because the local MEI equation has very small constant dimension, and there are almost the same number of MEI nodes as the number of panels in MoM, or BEM. So the memory and computation time of solving MEI equations are much less than solving the dense matrix in integral equation methods.

Although some papers[JL94] [JL95] propose some doubts on the third postulation of the MEI coefficients: invariant to excitations, they still admit in the papers that MEI is an efficient technique for the truncation of mesh boundaries. Actually, their arguments didn't conflict with the fundamentals of MEI, because we have already proven that MEI coefficients are actually not strictly invariant to excitations, but instead, are invariant to excitations on the sense of $O(h^2)$, where $h = max(h_x, h_y, h_z)$ [HSD95]. As stated above, the local FD equation Eq.6 to Eq.12 also has the error of $O(h^2)$, therefore, the total truncation/model error of the final matrix system has the order of $O(h^2)$, which is not degenerated by the introduction of MEI equations on boundaries. And because of the efficient absorbing property of the MEI coefficients, the final matrix system generated by MEI's method will approach the exact value with the convergence order of $O(h^2)$ when solved by using proper iteration methods. Up to now, the Measured Equation of Invariance has been successfully used to analyze electromagnetic scattering problems, and to analyze microwave integrated circuits [HLM94] [HM94].

However, the closed form Green's functions for multilayer structures of VLSI interconnects, are generally derived in spectral-domain and then transformed to the space-domain by inverse Fourier transformation which are infinite integrals. In addition to the tedious deduction of Green's function in a multilayer structure, the calculation of the MEI coefficients is very time-consuming because many Sommerfeld type integrals will be encountered. The calculation of MEI coefficients dominates the total computation time. As reported in [Pro94], for a one-layer microstrip stub, obtaining the MEI coefficients required 90 CPU minutes for a single frequency, and solving the sparse system required 24 minutes on a Dec Station 5000 series 200. Therefore, complicated Green's function and disagreeable Sommerfeld-type integrals make MEI very difficult, if not impossible, to be applied to multilayer and multiconductor interconnects.

4 Geometry Independent Measured Equation of Invariance

In order to overcome these drawbacks and apply this efficient truncation boundary concept in interconnects analysis, we introduced a measuring box concept. A measuring box is just a closed surface that encloses all objects inside as shown in Fig. 2 and Fig. 4, to isolate the MEI nodes(boundary and the next layer) and possibly some buffer layers from the region containing conductors. In [HSD96], it has been demonstrated that the MEI are also independent of the source distribution on the measuring loop. The MEI coefficients are then determined from the metrons on the measuring loop instead of the metrons on the conductors, which means the MEI are independent of the geometries of the conductors. In order to avoid the Green's function in multilayer structure, the dielectric layers are truncated at the measuring loop with physical polish which ensures such truncation will not affect the total accuracy, and free space out of the measuring loop is assumed. Therefore, we can use very simple free space Green's function to measure MEI coefficients. Experiments suggest that very few layer meshes between the measuring loop and the nearest conductors, and again very few layer meshes outside the measuring loop are sufficient to guarantee the accuracy of results in practice. The measuring loop concept has already been successfully applied to extract 2D parasitics of multilayer multiconductor interconnects.

Since free space is assumed out of the measuring loop, the potential values $\phi_i^k, i = 0, \ldots, M$, at the corresponding MEI nodes corresponding to kth metron σ^k defined on the measuring loop can be simply obtained:

$$\phi_i^k = \int_{\Gamma_e} \sigma^k(s')\, G(\vec{r}_i, \vec{r}\,')ds' \qquad i = 0, \ldots, M;\ k = 1, 2, \ldots, K \tag{14}$$

where Γ_e stands for the measuring loop, $\vec{r}_i, \vec{r}\,'$ denote the position vectors at ith MEI node and the measuring loop respectively, K is the number of metrons. The 3-D quasi-static Green's function of free space is simply

$$G(\vec{r}_i, \vec{r}\,') = \frac{1}{2\pi|\vec{r}_i - \vec{r}\,'|} - \frac{1}{2\pi|\vec{r}_i - \vec{r}\,''|} \tag{15}$$

where $\vec{r}\,''$ is the image position vector of $\vec{r}\,'$ with respect to the ground plane if any.

Substituting the potential values ϕ_i^k produced by kth metron into MEI (Eq. 13), yields

$$\sum_{i=0}^{M} C_i \phi_i^k = 0, \qquad k = 1, 2, \ldots, K \tag{16}$$

It is a system of linear algebraic equations with respect to the MEI coefficients $C_1, C_2, \ldots, C_M$, when C_0 is normalized to 1. If the number of equations or the number of metrons, is greater than M, we can solve Eq. 16 by least square techniques.

Generally, in three-dimension case, the point metrons are selected and clustering techniques are adopted. Because under qausi-static assumption, only the amplitude information(no phase information compared with full-wave approach) is needed in the determination of MEI coefficients, clustering is an efficient approximation. In our program, the CPU time to obtain MEI coefficients are much less than solving the final sparse matrix, which means the overhead time spent on MEI coefficients is only a very small part(less than five percents) in the total computing time.

Coupling the MEI equations at truncated mesh boundary nodes to the FD equations at interior nodes results in a matrix equation

$$[S]\, \bar{\phi} = \bar{f} \tag{17}$$

where $\bar{\phi}$ is a column matrix consisting of the potential values at all mesh nodes, and $\bar{f}$ is the known column matrix followed from the neighboring FD's around the conductors on which voltages are impressed.

From the solution of Eq. 17, we get the potential distribution over the mesh region. Since the finite difference approximation of the Laplace equation is less accurate in the vicinity of a conductor's reentrant corner(i.e., a corner whose outside angle is greater than π radians) because of a singularity in the electric field in the corner, we use Duncan correction [Dun67] to get charge distribution or total charge on each conductor. Bringing these charges into Eq.2, we can get the final short circuit capacitance matrix.

5 Experimental Results

As mentioned in the abstract, the new method proposed in this paper is very suitable to parameter extraction of general 3D interconnect structures. It has been demonstrated by experimental results that this method is faster than BEM(with multipole acceleration), MoM, and FD, without loss of accuracy. In addition, this method outperforms all methods for fairly large structures such as tens even hundreds of conductors on tens of dielectric layers. Furthermore, this method can be easily applied to structures with arbitrarily-shaped cross section conductors including infinitesimally thin conductors on lossy and inhomogeneous dielectric layers due to the nature of Finite Difference used inside the measuring loop.

To verify the accuracy and speedup advantage of this method, the following examples were selected to provide a quantitative measure. All relevant programs are run on a Sun Sparc 20 workstation.

5.1 Two dimension examples

Because the capacitance and inductance per unit length do not vary when we scale the whole geometry up or down in two-dimensional case, we only give relative size of each configuration without specifying the units in the following examples.

The first example we show is an infinitesimally thin microstrip as shown in Fig. 5. The characteristic impedance Z_0 of this structure can be defined as $Z_0 = \frac{1}{v_0\sqrt{CC_0}}$, where v_0 is the speed of light in free space, C the capacitance of this structure, and C_0 the capacitance with dielectric layer replaced by free space.

Fig. 5(b) shows the comparison of the characteristic impedance varying with the width height ratio W/H obtained by using GIMEI, Wei's result [CHMS84] which uses Method of Moments(MoM), Zutter's results [DZ89] which is based on Space Domain Green's Function Approach(SDGA), and those provided by Gupta [GGB79] and Hammerstad [hJ80]. In our results, we use ten mesh points per unit length. The difference of our results are within 2.5% compared with the results by Hammerstad [hJ80] which is regarded as standard reference for this kind of problem.

The second two-dimension example is three parallel wires immersed in a dielectric which is a commonly found structure in microelectronics, whose configuration is shown in Fig. 6(a). The structure represents three equidistant rectangular wires running parallel to a ground plane, where each conductor has the same size $w \times t$, and the space s varies from 5 to 60. This structure has been measured by Lin [Lin90].

cubes in meters	GIMEI results	FASTCAP results	difference percent	GIMEI CPU time	FASTCAP CPU time	CPU time $\frac{FASTCAP}{GIMEI}$
1x1x1	73.89	73.38	0.7%	0.21	0.7	3.3
1x1x3	114.8	115	0.2%	0.29	1.9	6.6
1x1x5	149.8	149.6	0.1%	0.34	3.6	10.6
1x1x8	196.4	196.2	0.1%	0.45	4.7	10.4
1x1x10	225.4	225	0.2%	0.52	6.8	13.1

Table 1: Self capacitances in pF of the cube and the CPU time in sec.

buffer number	capacitance(pF)	CPU time(sec.)	order of matrix
3	241.6	0.25	1,000
5	197	1.89	5,000
10	174	9.1	20,000
15	164	28.67	50,000
20	159.4	68.45	100,000
25	155	146.76	200,000
30	151	250.58	300,000

Table 2: Numerical results of $1 \times 1 \times 5$ meters cube using E.W. boundary condition

Fig. 6(b) shows capacitance of the middle conductor $C22$ varying with the interwire distance. A difference of less than 3% is observed in the whole range between our results and the measured data [Lin90]. It's clear from the figure that our results are more close to the measured data than those obtained by Finite Difference method(FD).

As stated in [HSD96], for larger examples such as 12 lines five dielectric layers, GIMEI is ten times faster than BEM with the difference within 3%.

5.2 A simple 3D example: cube with different longitude in air, compared with FASTCAP as well as FD with zero E field boundary condition

To verify the speedup and accuracy property of GIMEI, a simple example of $1 \times 1 \times z$ cube(unit in meters) is computed and compared with FASTCAP [NW92], which basically uses BEM and multipole acceleration. The cube computed is extended along one direction z. Table 1 shows the results(self capacitance of the cube varying with the extended edge z) of GIMEI compared with those of FASTCAP as well as the CPU time of the two programs run on Sun Sparc 20 workstation. From the table, one can clearly see that in general, GIMEI is generally ten times faster than FASTCAP with the difference of less than 1%.

We have also compared our results with those of standard FD with zero E field (electric wall, E.W.) boundary condition with the same mesh discretization as our method. The structure is chosen to be the $1 \times 1 \times 5$ meters cube. For our method, the buffers used outside the measuring loop is three mesh layers, the results is $149.8pF$ with 0.34 seconds CPU time. Table 2 shows the results by using E.W. varying with the layer number outside the measuring loop.

technique can also be extended to 2-D or 3-D dynamic analysis of multilayer multiconductor interconnect problems.

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