Visualizing Geometric Uncertainty of Surface Interpolants

Suresh Lodha; lodha@cse.ucsc.edu
Bob Sheehan; bob@cse.ucsc.edu
Alex Pang; pang@cse.ucsc.edu
Craig Wittenbrink; craig@cse.ucsc.edu

UCSC-CRL 95-46
October 29, 1995

Baskin Center for
Computer Engineering & Information Sciences
University of California, Santa Cruz
Santa Cruz, CA 95064 USA

ABSTRACT

Evaluating and comparing the quality of surface interpolants is an important problem in computer graphics, computer aided geometric design and scientific visualization. We introduce *geometric uncertainty* as a measure of interpolation error, level of confidence or quality of an interpolant. Geometric uncertainty can be estimated as a scalar or a vector-valued function that depends upon geometric characteristics of interpolants associated with the underlying data. These characteristics include position, normals, isophotes, principal curvatures and directions, mean and Gaussian curvatures. We present several new techniques for visualizing geometric uncertainty of surface interpolants, that combine the strengths of traditional techniques such as pseudo-coloring, differencing, overlay, and transparency with new glyph and texture-based techniques. The viewer can control an interactive query-driven toolbox to create a wide variety of graphics that allow probing of geometric information in useful and convenient ways. We demonstrate the effectiveness of these techniques by visualizing geometric uncertainty of surfaces obtained by different interpolation techniques – bilinear, C^0 linear, C^2 bicubic B-spline, multiquadrics, inverse multiquadrics and thin plate splines.

Keywords: comparison, geometry, glyphs, interactive, interpolation, probes, surfaces, uncertainty, visualization.

1. Introduction

Central to the work of scientists, engineers and designers is the task of constructing models of data sets obtained by instruments or created by users. However, in most situations, there is no clear choice of one model over another. Therefore, scientists, engineers and designers are very keenly interested in comparing the results from different models, and analyzing their relative advantages and disadvantages.

Data interpolation is one of the most important examples of this task. Franke compared several data interpolation techniques and evaluated the interpolants based on several characteristics such as accuracy, sensitivity to parameters and visual aspects [Fra82]. Mann et al. also compared several interpolants for triangulated scattered data in R^3 and evaluated the interpolants based on the shaded images of the interpolants or their Gaussian curvature plots [MLL$^+$92]. Effective display of geometric information associated with surface interpolants has become an important tool in evaluating and comparing the quality of surface interpolants in computer graphics, computer aided geometric design and scientific visualization.

We introduce the term *geometric uncertainty* as a measure of interpolation error, level of confidence or quality of an interpolant. Geometric uncertainty can be estimated as a scalar or a vector-valued function that depends upon geometric characteristics of interpolants associated with the underlying data. These characteristics include position, normals, isophotes, principal curvatures and directions, and mean and Gaussian curvatures. Other measures of geometric uncertainty will be discussed in Section 2.1.

Visualizing geometric uncertainty is a very valuable aid in evaluating the effectiveness of an interpolation scheme. Side-by-side display of interpolants or some geometric property of interpolants such as Gaussian curvature is perhaps the most popular technique for comparing interpolants. Other popular techniques include pseudo-coloring, differencing, overlay and animation. Although these techniques have been found to be successful to some extent, no one technique is flexible or powerful enough to provide the wide range of information that a user typically seeks. Moreover, most of the past methods provide no control to the user for probing the quality or geometry of the interpolants.

In this work we present new techniques for visualizing geometric uncertainty of surface interpolants. There are two major strengths of the system that we have designed. First, we have used a wide range of visualization techniques that combine the advantages of traditional techniques with new glyph-based techniques that capture the geometric information through shape, size and color of glyphs. We have also incorporated texture-based visualization techniques that include bump mapping, displacement mapping and spot mapping. Second, our system provides an interactive control to the user for probing geometric information of surface interpolants in many useful and convenient ways. Examples include displaying only a subregion of interest that satisfy certain constraints, or mapping geometric information of interest to visual objects such as glyphs, or manipulating characteristics such as width or display resolution of glyphs in order to create graphics that are convenient to view.

In order to demonstrate the effectiveness of our techniques, we have implemented several interpolation schemes that include multiquadrics, inverse multiquadrics and thin plate splines. We have also implemented bilinear interpolation and C^2 bicubic B-spline interpolation schemes that work only on gridded data. The geometric uncertainty information of these interpolants has been computed and visualized. Experimentation with the visualization techniques brings out a wealth of information about the interpolants in a convenient and effective manner.

The rest of the paper is organized as follows. Section 2 describes previous work on defining and visualizing uncertainty in general and geometric uncertainty in particular. Section 3 presents new techniques for visualizing geometric uncertainty. Section 3 also describes interactive features of the system that allow the user to probe the interpolants effectively and conveniently. Section 4 presents the implementation and discusses the results of our experimentation. Section 5 presents two experiments on visualizing the geometric uncertainty of surface interpolants. Section 6 concludes with final remarks and future work.

2. Background

In this section we describe the previous work on defining and visualizing uncertainty with an emphasis on geometric uncertainty.

2.1 Uncertainty

Uncertainty is a term that has been used to describe several different features of scientific data including error, accuracy, confidence level and quality of data. Error can be defined as the discrepancy between a given value and its true value [GBW94]. Inaccuracy is the difference between the given value and its modeled or simulated value [GBW94]. Confidence level is the level of confidence that can be associated with data and can be computed based on statistical methods or evaluation by scientific judgement [TK93]. Data quality is a very broad term that encompasses many concepts including data validity and data lineage [BBC91, Moe88].

Geometric uncertainty, likewise, is a scalar or a vector-valued function that captures error, accuracy, quality or confidence level of the geometry of a surface. The geometric characteristics of interest typically include several pieces of geometric information that are based on positional, first, second and sometimes even third derivative information. The first derivative information of interest at a point on the surface includes tangent plane information, normals and isophotes. Given a normal $\vec{N}(p)$ at the point p on a surface and a direction $\vec{L}$ of the light source, the *isophote* surface $I_{\vec{L}}(p)$ can be defined as $I_{\vec{L}}(p) = \vec{N}(p) \cdot \vec{L}$, where $\cdot$ denotes the dot product. There is a continuum of isophote surfaces depending upon the direction of the light source. Contours of isophote surfaces have been used to interrogate surface geometry [HHS$^+$92]. Most of the geometric measures that capture second derivative information are based on minimum and maximum principal curvatures κ_1 and κ_2 and the associated principal directions $\vec{e_1}$ and $\vec{e_2}$ respectively. We refer the reader to any standard textbook on differential geometry for details [dC76]. Important geometric measures for surfaces are Gaussian curvature $K = \kappa_1\kappa_2$ and mean curvature $H = \frac{1}{2}(\kappa_1 + \kappa_2)$. Both Gaussian and mean curvatures are geometric invariants that capture the local geometry of the surface. The quantity $\kappa_1^2 + \kappa_2^2$ measures the strain energy of flexure and torsion in a thin rectangular elastic plate with small deflection, and is typically used as a standard fairness criterion for surfaces in engineering [HS91]. Third derivative information is captured by the sum of the variations of the principal curvatures along the principal directions, that is, $(\frac{d\kappa_1}{d\vec{e_1}})^2 + (\frac{d\kappa_2}{d\vec{e_2}})^2$, which has also been used as a fairness metric [MS94]. Other more sophisticated criteria have also been adopted [HB93, MS94]. In addition, reflection lines, orthotomics and focal surfaces have also been proposed for surface interrogation [HHS$^+$92]. In principal, any of the above measures or weighted combination of these measures or differences between these measures can be used as an estimate of geometric uncertainty. The exact choice depends upon the application at hand.

2.2 Visualizing Uncertainty

Popular techniques for visually comparing surface interpolants are side-by-side comparisons, difference comparison and pseudo-coloring. Franke compared visual aspects of several interpolants by drawing wireframe perspective plots side-by-side [Fra82]. Isophotes have been compared by drawing the contours of isophote surfaces side-by-side [HHS$^+$92, PHD91]. Examples of side-by-side comparison also occur in comparing 2D images after wavelet compression [DJL92] and comparing 2D images of 3D volumetric data after hierarchical volume rendering and compression [WG94]. Difference comparison is a technique where the difference between two images, surfaces or volumes is computed point-by-point and the difference image, surface or volume is rendered. Examples of this occur in comparing images by Tvedt [Tve91] and comparing volumes by Foley et al [FLN90]. Pseudo-coloring has been used to compare Gaussian curvature of surface interpolants by Lounsbery et al. [LMD92].

Other techniques for visual comparisons include transparency, overlay and animation. Use of transparency for comparing surface interpolants is presented in [PFN94]. Related concepts of blends (including techniques based on percentage classification of materials), fuzziness, fog or blurs have been proposed in [FLN90, BBC91]. The idea of overlaying two curves or surfaces and connecting the respective points by straight lines has also been used [LSG94]. Animation has been used to visualize fuzzy data [Ger92].

Although glyphs or textures have not been used for comparing or visualizing surface interpolants, they are quite common in data displays. Glyphs are symbols that represent data through visual properties such as size, shape, color, position and orientation. They have also been called probes, geometrical primitives, stars, boxes and icons [PG88]. Glyphs have been used to represent univariate data [Tuk84, Tuf83b, Tuf83a, Cle85]. Different types of glyphs such as stars, Chernoff faces, boxes, profiles, Kleiner-Hartigan tress and Andrew's plots have been used to represent multivariate data [CBB91]. Glyphs for representing vector and tensor fields are shown in [dLvW93]. Texture mapping has been used for generating photo-realistic images [Hec86] and scientific visualization [vW91]. Displacement mapping and bump mapping are also standard techniques in computer graphics [FvDFH90].

In addition to the techniques mentioned above, most of the work in visualization of uncertainty has been in the field of Geographic Information Systems, for which we refer the reader to [HG93] or [WSF$^+$95]. We also mention that several techniques have been proposed for visualizing surfaces over surfaces and multi-valued volumetric visualization [FL90, FL91, Nie87, NFHL91], but none of them seems to have addressed the question of visually *comparing* surfaces or visualizing geometric uncertainty. Finally, visual comparison of sequences also have been studied [HW91].

3. Features of the System

We now present an overview of our system for visualizing geometric uncertainty of surface interpolants and the key factors that influenced the design of the system. First, although traditional visualization techniques such as pseudo-coloring or differencing have been successful to some extent, no one technique is flexible or powerful enough to provide the wide range of information that a user typically seeks. Therefore, our system creates a wide range of visualization possibilities that incorporate the complementary advantages of different visualization techniques. Second, in our visualizations, we have attempted to incorporate the important principles of data-ink maximization [Tuf83b] and maximum impact [Tuk84] by providing a clutter-free presentation and focusing on the substance of the presentation. More importantly, we are guided by the principle of maximum utility to the user. Therefore, the user is provided with an interactive query-driven toolbox that allows the facility to control many parameters such as geometric uncertainty parameters, subregion selection, scaling, lighting, zooming, translation, rotation, color ramps to create their own views. Moreover, in our visualizations, we have included many retinal or visual variables such as shape, size, and color based on Bertin's classification [Ber83]. We now discuss both these features in greater detail.

3.1 Visualization Techniques

In order to capture diverse geometric information together, we have created visualizations based on *geometry glyphs*. Geometry glyphs are visual objects that convey geometry through its visual properties such as size, shape, color and position. The user can choose between many different shapes that include boxes, spheres and ellipsoids. Shapes, sizes and colors can be mapped to user-preferred geometric parameters. These choices provide a wide range of possible glyphs. We now describe specific examples of some glyphs that we have found useful. A *displacement glyph* (Figures 3.5 and 3.2) at a point is a thick line or a cylinder or an ellipse or a box, the height of which encodes the geometric information of interest at that point. A *cross-hair glyph* (Figures 4.1, 4.2 and 4.3) consists of two orthogonal planes, the heights of which encode uncertainty of mean and Gaussian curvatures. A *triangular glyph* (Figures 3.4 and 4.2) is a vector-glyph that displays the triangular region between two vectors at the same point. We have used triangular glyphs to display the geometric uncertainty of normals and principal curvature directions at a point. We have also created a *volume-filling glyph* (Figure 3.1) that encloses the volume between two surfaces by spheres whose radii are proportional to the difference between two surfaces.

In order to create visualizations that are clutter-free and easy to perceive, we have used texture mapping for capturing geometric uncertainty information. Three different techniques

Figure 3.1: Volume filling glyphs between multiquadric (MQ) and thin plate spline (TPS) interpolants

Figure 3.2: Swept probes along a selected triangle for MQ interpolant with displacement glyphs and pseudo-coloring mapped to the difference between the MQ and TPS interpolants

Figure 3.3: Displacement mapping for C^2 bicubic B-spline interpolant with displacement randomly proportional to the difference between this interpolant and the bilinear interpolant

of texture mapping have been implemented and investigated: displacement mapping, bump mapping and spot mapping. In displacement mapping (Figure 3.3), one of the surfaces is randomly perturbed in proportion to the geometric uncertainty parameter. In bump mapping, the normals to the surfaces are perturbed. In spot mapping, regions of high relative differences appear spotted (Figure 3.4). The spot texture or jitter created in the surface highlights the regions of interest without extra gadgets as with glyphs.

Our visualization system also incorporates most of the traditional visualization techniques including side-by-side comparisons, pseudo-coloring (Figures 3.2 and 3.6), differencing (Figures 3.6 and 4.3), overlay (Figure 3.5), animation and transparency (Figures 4.2 and 4.3) for visualizing any one geometric feature of interest. Our contribution here is to allow the user to choose from a wide variety of geometric uncertainty parameters, described in Section 2.1.

Combinations of these techniques provide a richer and more useful class of techniques. For example, overlay surfaces can be combined with displacement glyphs (Figure 3.5), difference surface can be pseudo-colored (Figure 3.6), or cross-hair and triangular glyphs can be used with transparency (Figure 4.2). By combining these techniques judiciously, we have created a wide range of new possibilities for probing the geometry of surfaces. Advantages of these visualization techniques are presented in Section 4.

3.2 Interactive Features

This visualization system provides the user with query-driven interactive control of several features in order to create graphics that are useful and convenient to view.

Visual Parameter Selection: With every visualization technique, there are several visual parameters that can be controlled by the user. In glyph-based techniques the user can choose the display resolution as well as the size, shape and color of the glyphs. In texture-based techniques, the user can choose the randomness factor. In transparency or pseudo-coloring, the amount of transparency or the choice of the color ramp is up to the user. In addition, there are several visual parameters that are not tied to any particular visualization technique. For example, the user can position the lights, choose the intensity and colors of the light and choose material properties of the surface such as the coefficients of reflectivity for ambient, diffuse and spectral light. The user also has the ability to view a wireframe representation or a shaded representation. This flexibility can be used for three different purposes:

Figure 3.4: Spot texture mapping with triangular strips indicating uncertainty in normals above a certain threshold for MQ and TPS interpolants

1. *To create views that are easy to navigate and understand*: This objective is achieved by mapping visual parameters according to convenience of viewing. For example, the display resolution can be chosen for a dense (Figure 4.1) or a sparse presentation (Figure 4.2). Size of the glyphs have been scaled in Figure 4.3 because the original glyphs were too small to view indicating that the absolute differences between the two interpolants are very small. A green-red ramp is chosen in Figures 3.2 and 3.6 over a standard grey ramp, because it indicates not only the magnitude of the differences between the two surfaces by the brightness, but also the sign of the differences by the color. The amount of transparency has been manipulated in Figures 4.2 and 4.3 to display a transparent surface where the differences are small and relatively opaque where the differences are large. The randomness factor in displacement mapping has been chosen in Figure 3.3 to present a certain level of contrast that is meant to represent the level of confidence in the interpolant. Regions of low level of confidence appear uncertain due to its rough texture.
2. *To overload an image with additional cues:* Visual parameters are mapped to the same geometric information in order to reinforce the data with different visualization techniques. Figure 3.5 displays an isophote of the multiquadric interpolant in green and the corresponding isophote of the thin plate spline interpolant in red. The differences between the two isophotes are then filled in by displacement glyphs. Both the mappings – overlay and the displacement glyphs – encode the same information about the position of the isophotes. However displacement glyphs provide additional cues. As another example, Figure 3.2 displays a surface that has been pseudo-colored according to the difference between the two interpolants in addition to the glyphs that encode the same information through their heights. Both the mappings – the pseudo-color and the glyphs – provide the same information but reinforce each other in a strong way to provide a much better understanding of both relative and absolute values.
3. *To create a single graphic that brings together diverse geometric information together:* In order to achieve this objective, visual parameters such as glyph parameters, texture parameters, amount of transparency or the color ramp are mapped to different geometric uncertainty parameters. Figure 4.2 displays the multiquadric interpolant, where differences between the multiquadric and the thin plate spline interpolant are highlighted using transparency technique, differences in normals are shown by triangular strips and cross-hair glyphs have been utilized to display the differences in mean and Gaussian curvatures. This graphic combines the positional, the first derivative and the second derivative uncertainty information in a single graphic.

Query-Based: This refers to the ability of the user to highlight or display only a part of the entire graphic that satisfies certain constraints or queries. These queries are tied to the geometric properties of the surface. An example of such a query is to display only those glyphs that represent large differences between normals (Figure 3.4) or represent differences between

Figure 3.5: Wireframe overlay of an isophote of MQ and TPS interpolant with displacement glyphs reemphasizing differences in the corresponding isophotes

Figure 3.6: Difference of Gaussian curvatures of MQ and TPS interpolant with pseudo-coloring mapped to the difference between the two interpolants

Gaussian curvatures within a certain range. This facility is important in several situations. An example is when small differences may clutter the presentation and the viewer may want to remove them. Another example is when large differences dominate in a pseudo-colored view and the user wants to remove them in order to focus on regions with intermediate or low values.

Region Selection: This refers to the ability of the user to select certain subregions of interest. For example, the viewer can choose to view only the region around a hill or a saddle point. Our system provides the facility to the user for viewing only that part of graphics that are associated with a curve or a point. The user can select these subregions either by clicking with a mouse or by providing the location of the point or the equation of the curve. This feature is useful for probing the surface at a given point, surrounding regions or along boundary curves. Glyphs along the curves can be animated with *animated probes.* In this case a glyph such as an ellipsoidal ball or a box moves along a curve on one surface and expands or shrinks according to the difference between two surfaces along that curve. The user can control the speed of the probe. Alternatively, the glyphs along the curves can be swept along a desired curve and retained for subsequent viewing in *swept probes* (Figure 3.2).

The system allows standard geometric and viewing transformations such as translation, scaling, rotation and zooming. We also have a 3D-trackball that allows user to pick a direction of the light source interactively in order to create an isophote surface.

4. Implementation and Analysis

We now describe the interpolation schemes and data sets used in the experimentation of our visualization system. We then discuss the results of our experiments.

4.1 Interpolants

We have implemented several interpolation techniques, that are quite popular in computer graphics, computer aided geometric design and scientific visualization applications. These interpolants are C^0 piecewise linear interpolant (based on a triangulation of the data), bilinear interpolant, and C^2 bicubic B-spline interpolant for gridded data. For the bicubic B-spline interpolants, we have used the generalization of not-a-knot boundary condition [Wol90] for constructing tensor-product interpolants. We have also implemented Hardy's multiquadrics, inverse multiquadrics, and thin plate splines. The motivation for choosing these radial interpolants is that these three radial interpolants are the only ones (besides one more radial interpolant) that received an 'A' rating in visual category in Franke's survey [Fra82].

4.2 Examples and Data Sets

We have experimented with Franke's six analytic test functions [Fra82], which include a wide variety of shapes including hills, valleys, cliffs, saddles and a part of a sphere. The equations for these functions are available in [Nie87]. We have set the value of the free parameter for multiquadrics and inverse multiquadrics interpolants for Franke's test functions to be the one reported by Foley et al. [Fol94], which is nearly optimal for a slightly different distribution of data. For each of these functions, the interpolants can be constructed by sampling the analytic functions for different data distributions [Nie87]. We have also experimented with some meteorological and oceanographic data obtained by instruments. Due to limited space, in this paper all the figures correspond to interpolants constructed by sampling Franke's first analytic function (that contains two hills, a valley and a saddle), on a 10×10 grid. Geometric uncertainty in these figures is computed as the difference between the geometric quantity of the two interpolants. For example, in Figure 4.2 the height of the cross-hairs depict the difference between the mean curvatures (in red) and the Gaussian curvatures (in green) of the two interpolants.

Figure 4.1: Mean curvature of MQ interpolant with cross-hairs displaying differences between the mean curvatures of MQ and TPS interpolants in red and the differences between Gaussian curvatures of MQ and TPS interpolants in green

Figure 4.2: MQ and TPS interpolants using transparency; triangular strips indicating uncertainty in normals; cross-hairs displaying uncertainty in mean and Gaussian curvatures

4.3 Discussion

We now discuss the results of our experimentation with visualizing geometric uncertainty. The key observation is that a static visualization system is highly constrained to be of much value in a practical situation. The key to a successful system is providing flexibility in creating visualizations by possible combinations of (i) visualization techniques, (ii) geometric uncertainty parameters, and (iii) visual parameters. This flexibility was heavily utilized in creating examples of visualizations presented in this paper and in conducting the experiments for probing the quality of surface interpolants described in Section 5. Examples and advantages of flexibility in choosing visual parameters are described in Section 3.2. Here we focus on analyzing the advantages and disadvantages of different techniques for visualizing geometric uncertainty.

Glyphs: We have found both the displacement glyphs and volume filling glyphs to be one of the most useful and precise techniques for comparing surfaces visually. Displacement glyphs give a very good idea of absolute differences between surfaces. They also provide the information as to where these differences are located as well as the relative positions of the two surfaces. Volume filling glyphs are very useful in providing a good sense of the error by filling the total volume enclosed between the two surfaces. Even if the absolute differences are rather small, this method can be made very effective by scaling the glyphs, by choosing different glyph shapes, by adjusting the spacing between glyphs and by zooming into the areas of interest. For example, spheres are better than boxes for small differences but worse for large differences because they tend to bulge out.

Texture Mapping: Displacement mapping, bump mapping and spot mapping provide relatively easy to view information about the regions where the two surfaces disagree. Although these methods seem to do a crude job of providing precise quantitative information, they are very effective both as additional cues and in having a clutter-free presentation even after adding more information about an additional geometric feature.

Transparency: Transparency uses much less data-ink to portray the same information and is very helpful in providing clutter-free presentation. This technique is also useful due to its see-through mechanism. However, this method does not provide a precise idea of absolute differences between the two quantities.

Difference Surface: This method is very effective in assessing the absolute difference between two quantities. By scaling, this method can also bring out regions of high relative differences. The location of these differences can also be grasped very easily relative to the domain, but not with respect to the range.

Overlays: Overlays provide satisfactory information about the relative placement of two surfaces or the two geometric quantities. However they are rather difficult to view due to intersections between two surfaces.

Figure 4.3: Difference between an isophote of MQ and Inverse MQ interpolants using transparency; cross-hairs displaying uncertainty in mean and Gaussian curvatures

Pseudo-color: Pseudo-coloring technique is effective in bringing out the regions of high relative differences. However it is difficult to gain good understanding of the absolute value of the differences using this method.

Animation: We found it rather difficult to get much useful information from a simple animation between two surfaces. However when combined with animated probes that expand in proportion to differences between surfaces along prescribed curves over which they move, they become an effective method for detailed information in regions of interest.

Side-by-side comparison: This method is effective in revealing large structural differences only when they exist. However the eye cannot detect many subtle and even intermediate scale differences particularly when the differences are shifts of similar features.

Systematic usage of the variations and combinations of these techniques yields a wealth of information, that is not available when restricting oneself to only one variation or technique. We mention only a few examples. Overlaid surfaces along with displacement glyphs (Figure 3.5) provide the user with a much better understanding (in an interactive mode) of both the relative positions of two surfaces as well as the magnitude of the differences between the two surfaces, and overcomes the difficulties encountered by other popular techniques acting alone such as side-by-side comparison, difference surfaces and pseudo-coloring. Glyphs with pseudo-color provide both absolute and relative difference information relative to the features of the surfaces (Figure 3.2). Difference surface with transparency provides both relative and absolute difference information on the domain (Figure 4.3).

5. Applications

We now describe two experiments for probing the surface geometry of interpolants using visualization techniques developed in this work.

5.1 Experiment 1

This experiment describes comparisons of multiquadric (MQ) interpolant with the thin plate spline (TPS) interpolant for the data set mentioned in the previous section. Both these interpolants were assigned an 'A' rating in visual aspects by Franke [Fra82]. We wanted to probe the geometry of these interpolants in order to make finer distinctions between these two interpolants.

The data set has two hills in the back (in still views displayed in this paper), a saddle between the two hills and a valley in the front. We first compared the two interpolants by looking at the pseudo-color. This visualization indicated that the differences between the two interpolants are relatively worst at the valley followed by the two hills and near the saddle. This observation was reaffirmed by transparency technique. Both these techniques however failed to give an idea of the absolute difference between the two interpolants. This was easily assessed by looking at the difference surface and even more effectively by volume-filling glyphs shown in Figure 3.1. We also observed that the MQ interpolant was a better fit than the TPS interpolant by reaching higher (and closer to the true analytic value) at the two hills and by dipping lower (and closer to the true analytic value) at the valley by comparing both the multiquadric and the thin plate spline interpolant with the analytic surface. This observation was again reaffirmed by comparing other features such as Gaussian and mean curvatures of the MQ and TPS interpolants with the same features of the analytic surface.

To visualize the uncertainty in normals, we used triangular strips that displayed the differences between the normals of the two interpolants. Figure 3.4 shows the displacement of the two interpolants as a spotted texture while only the differences between normals within a certain range are shown in this figure. Other than the large differences in normals at the hills and valleys (which are suppressed in this figure), the normals are deviant in the flatter regions (on the right of the valley for example). This observation was reconfirmed by visualizing a series of corresponding isophotes of two surface interpolants. Figure 3.5 shows isophotes of the multiquadric and thin plate spline interpolants overlaid over each other. The differences in the isophotes shown as displacement glyphs are thresholded and are more pronounced in flatter regions.

We then compared the Gaussian curvature information of two interpolants. First, the two hills and a valley correspond to three hills in the Gaussian curvature plots for the multiquadric interpolant. More significantly, the saddle point, which remains relatively unattractive to the eye in displays of interpolants, becomes an important feature as a valley in the Gaussian curvature plot of the multiquadric interpolant. Performance of the Gaussian curvature plot of the thin plate spline was observed to be rather poor. Figure 3.6 shows the difference between the Gaussian curvature of the two interpolants. Pseudo-coloring is mapped to the difference between the two interpolants. The interesting observation here is the phenomena of a steep hill adjacent to a steep valley at all the three hills of the Gaussian curvature. This phenomenon is visible in the front in the image shown in 3.6 near the valley. Two more occurrences of this phenomena are near the two hills at the back.

Figure 4.1 shows the mean curvature of the MQ interpolant. Here the saddle point appears as a ridge due to approximate cancellation of the two principal curvatures. The dense cross-hairs in this figure clearly bring out those red regions where differences between mean curvatures dominate as compared to green areas where differences between Gaussian curvatures dominate. Finally, Figure 4.2 compares the positional, first and second derivative information in a single graphic, that is clutter-free and easy to navigate to obtain additional and precise information.

5.2 Experiment 2

This experiment compares multiquadric and inverse multiquadric interpolant for the same data set. Both these interpolants do an excellent job of fitting this data set. In fact the difference surface is essentially flat and the eye can hardly capture any difference. Pseudo-coloring and transparency emphasize relative difference and fail to provide much meaningful information. We wanted to investigate if we can discover anything further about these two interpolants using visualization techniques developed in this work. We experimented with isophotes and normals to bring out the most significant differences between the two interpolants. The differences between normals were again mostly insignificant for the eye to detect. The absolute differences between the corresponding isophotes also remained small for almost all directions including the one shown in Figure 4.3. Figure 4.3 shows the transparent surface that displays the difference surface between corresponding isophotes of the two interpolants for a chosen direction of light. The regions of high relative differences in isophote surfaces are brighter.

We then investigated the differences between Gaussian curvature and mean curvatures of the two interpolants. These differences were also rather small. In order to bring out the regions of high relative differences, we mapped the curvature difference information and scaled the glyphs manyfold. Then by thresholding the low differences in curvatures, small green patches (where the two hills and the saddle point at the back and the valley in the front are located) in Figure 4.3 brings out that the two interpolants differ relatively more in Gaussian curvature at these features while the differences in mean curvature are more significant in flatter regions on both sides of the valley. By comparing the Gaussian and mean curvature of the interpolants with those of the analytic surface, it became clear that the multiquadric interpolant does a slightly better job than the inverse multiquadric interpolant in the valley region.

6. Conclusions

In this work, we have described several techniques of visualizing geometric uncertainty of surfaces. The user can create a wide variety of visualizations by choosing appropriate combinations of visualization techniques and geometric features of interest. The user is also able to perform interactive queries, select subregions of interest and map a variety of visual parameters in order to create useful and effective graphics. The system was applied to probe the geometry of surface interpolants and revealed wealth of information conveniently and quickly.

Visualization techniques developed in this work can be applied to data assimilation, that is for comparing and correlating data from models and observations. There is also a great need for extensive experimentation in order to evaluate and assess the usefulness of different techniques in specific application domains such as visual comparisons of different radiosity techniques. We are actively investigating these applications. Techniques developed in this work are also applicable to more general settings. Potential applications include comparing images obtained by different compression methods, comparing textures obtained by different warping schemes, and comparing volumetric images created by different rendering methods.

Acknowledgments: This work was partially supported by National Science Foundation grants IRI-9423881, CCR-9309738 and CDA-9115268, ONR grant N00014-92-J-1807, and by the faculty research funds granted by the University of California, Santa Cruz.

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