

Transient Analysis of Coupled Transmission Lines Characterized with the Frequency-Dependent Losses Using Scattering-Parameter Based Macromodel

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ABSTRACT

This paper describes two novel macromodels for incorporating the single and coupled transmission Lines characterized by the frequency-dependent losses into an S-parameter macromodel based simulator. This approach computes the moments of the S-parameter based upon the frequency-dependent parasitic functions: $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$, and $\mathbf{G}(f)$ which characterize either the single or the coupled transmission lines. These same moments can be used later to construct the macromodels. Once the macromodels are built, the transient analysis can be performed by using the Scattering-Parameter (S-parameter) based macromodel simulator.

Keywords: Coupled Transmission Lines, Transient Analysis, Congruence Transformation, Scattering Parameter, Macromodel, S-Parameter Based Simulator, Measured Data, Frequency-Dependent Losses

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1 Introduction

Interconnect design has become an important issue in the high performance systems due to the recent advance in the processing technology. As the packaging density and the clock speed of the Integrated Circuit (IC) increase, the frequency-dependent losses as well as the crosstalk induced noise have become some of the major issues in the designs of the interconnect networks. As a result, an accurate and a more efficient circuit simulator is necessary for the design of the more advance ICs.

Due to the high switching speed of today's digital systems, the magnitudes of the harmonics of the transmitted signals above 1 GHz can often become the significant components in the power spectrum. At these high frequencies, the interconnects exhibit more frequency-dependent conductor (skin effect) and the dielectric losses. The electro-static field solution can no longer predict the correct parasitics for the interconnects that carry the high-speed digital signals. The macromodel of the transmission lines is required to accurately model both the frequency-dependent conductor (skin effect) and the dielectric losses.

The objectives of this paper is to provide a method of finding the Taylor series expansions of the S-parameter functions from the measured parasitic data, $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$ and $\mathbf{G}(f)$. A curve-fitting is first applied to find the moments of these four parasitic functions, which are used later to compute the exact moments of the S-parameter functions. Based on the exact moments found using this approach, the macromodel of transmission lines characterized with the frequency-dependent losses is constructed and the transient simulation is performed.

The scattering parameter (S-parameter) based macromodel simulator has been previously developed as a novel circuit simulator. Given the scattering parameter description of the measured data, lumped elements, interconnect junctions [17], and single transmission lines, combining with the use of the two efficient reduction rules, the original distributed and lumped network can be reduced by the circuit simulator into a network containing one multi-port component together with the sources and the loads of interest [19]. In addition, when the lower order approximation is used in the representation of the scattering parameter macromodels, a better control of a trade-off between accuracy and efficiency of the transient simulation can be obtained. Their utility, however, is very limited due to the number of macromodels available because of the relative short course of its existence. It is therefore important to pursue other macromodels that can deal with the crosstalk noise and the

frequency-dependent issues as well as devices characterized with the measured S-parameter data. In the following sections, two newly developed macromodels will be presented which represent the frequency-dependent single and coupled transmission lines.

Compared to other accurate time domain simulators, the S-parameter macromodel based simulator provides efficiency because it is at least thirty times faster. However, it only provides moderate accuracy because it utilizes lower order Pade approximation. This trade-off between accuracy and efficiency has to be made in order to play what-if scenario for a performance-driven layout synthesis which thousands of simulations must be executed to obtain timing and amplitude information. Traditional approaches which use empirical equations might not work during performance-driven layout synthesis because there are a lot of assumptions made by them. Only a simulator with enough accuracy yet does not take a long time to simulate can fit this requirement. The S-parameter macromodel simulator is well suitable for this kind of application.

The S-parameter macromodel of the frequency-dependent single and coupled transmission lines will be discussed in Section 2. The experimental results will be presented in Section 3 and the conclusion will be addressed in Section 4.

2 Representing the Frequency-Dependent Coupled Lines Using the S-Parameter Macromodel

Previous researches which took frequency-dependent losses into consideration include those by Gruodis et al. [14], Schutt-Aine et al. [24], Chang et al. [6], Beyene [3], Baumgartner [1], Cooke et al. [9], Gordon et al. [13], and Nguyen [21]. Gruodis et al. measured the admittance matrix \mathbf{Y}_{2n} and impedance matrix \mathbf{Z}_{2n} of the transmission lines, computed the \mathbf{Y}_0 matrix and propagation constant Γ , and then simulated the circuit's transient behavior using the state variable transfer function method [14]. Schutt-Aine et al. utilized the scattering parameter matrix method [24]. Chang et al. chose the method of characteristic with a network synthesis [6]. Beyene combined the bi-level waveform relaxation with scattering parameters [3]. Baumgartner used a state variable transfer function with an exponential approximation [1]. Cooke et al. selected to use the scattering parameter frequency domain simulation with the Fast Fourier Transformation (FFT) method [9]. Gordon et al. used the impulse response convolution method [13], and Nguyen preferred to use the state variable transfer function with the rational function approximation improvement [21]. In

some of these previous researches including those by Schutt-Aine et al. [24], Beyene [3], and Cooke et al. [9], they chose to use the scattering parameter for the analysis of the transmission lines with the frequency-dependent losses. The scattering parameter matrix method adopted by Schutt-Aine et al. [24] used those of $S_{11}(s)$ and $S_{21}(s)$ to find the transmission $T(t)$ and reflection $\Gamma(t)$ matrix for the time domain convolution. The drawback of this Schutt-Aine method is the large number of matrix operations that were required. The bi-level waveform relaxation method adopted by Beyene [3] utilized the FFT and the inverse FFT (IFFT) to iterate between the frequency domain simulation and the time domain simulation. The deficiency of Beyene's method is the need of more than one thousand data points in order to do the evaluation of the FFT (IFFT) operation with the same degree of accuracy as comparing to other approaches. The scattering parameter frequency domain simulation with the FFT method by Cooke et al. [9] selected to use all the scattering parameters without reduction. The drawbacks of Cooke et al. method are its poor efficiency and its lack of accuracy when compared to the published ASTAP results. In summary, all of the above methods lack the required efficiency because they do not employ the lower order approximation and the macromodel reductions that were found in the scattering parameter macromodel based simulator.

Building the novel macromodels for both the frequency-dependent single and coupled transmission lines facilitates an accurate and a more efficient transient simulation of the interconnections that are characterized by lossy transmission lines with skin effects. The contribution of this part of the paper is to determine the moments of the S-parameters of the decoupling congruence transformer and the decoupled transmission lines characterized with frequency-dependent losses from the curve-fitting coefficients of the $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$ and $\mathbf{G}(f)$ data sets. Section 2.1 will derive the representations of the S-parameters macromodel for the frequency-dependent single transmission line and Section 2.2 will derive the representations of the S-parameters macromodel for the frequency-dependent coupled transmission line.

2.1 S-Parameter Macromodel of the Frequency-Dependent Single Transmission Line

With the assumption of quasi-TEM wave propagation, the distributions of voltages and currents in a single lossy transmission line can be described by the generalized Telegraphist's equations [7]:

$$\frac{\partial \mathbf{v}(x, t)}{\partial x} = -\mathbf{L}(f) \frac{\partial \mathbf{i}(x, t)}{\partial t} - \mathbf{R}(f) \mathbf{i}(x, t) \quad (2.1)$$

$$\frac{\partial \mathbf{i}(x, t)}{\partial x} = -\mathbf{C}(f) \frac{\partial \mathbf{v}(x, t)}{\partial t} - \mathbf{G}(f) \mathbf{v}(x, t), \quad (2.2)$$

where $0 \leq x \leq l$, and $\mathbf{v}(x, t)$ and $\mathbf{i}(x, t)$ define the voltage distributions $v_k(x, t)$ and current distributions $i_k(x, t)$ on the conductor. The $\mathbf{L}(f)$ and $\mathbf{C}(f)$ are the frequency-dependent per-unit-length inductance and capacitance of the single conductor system. The $\mathbf{R}(f)$ is the frequency-dependent per-unit-length resistance of the single conductor. The $\mathbf{G}(f)$ is the frequency-dependent per-unit-length conductance of the single conductor.

The $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$ and $\mathbf{G}(f)$ are used to characterize transmission line with frequency-dependent losses. The scattering parameter matrix for a transmission line is shown in Equation (2.3).

$$S(s) = \frac{1}{2Z_0 Z_c(s) \cosh(\gamma(s)) + (Z_c^2(s) + Z_0^2) \sinh(\gamma(s))} \begin{bmatrix} (Z_c^2(s) - Z_0^2) \sinh(\gamma(s)) & 2Z_0 Z_c(s) \\ 2Z_0 Z_c(s) & (Z_c^2(s) - Z_0^2) \sinh(\gamma(s)) \end{bmatrix}, \quad (2.3)$$

The $Z_c(s)$ is the characteristic impedance and $\gamma(s)$ is the propagation constant. Both $Z_c(s)$ and $\gamma(s)$ are computed from the frequency-dependent values of $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$ and $\mathbf{G}(f)$ based on the following Equations:

$$\begin{aligned} \gamma(s) &= \sqrt{(R(s) + sL(s))(G(s) + sC(s))} \cdot l \\ Z_c(s) &= \sqrt{\frac{R(s) + sL(s)}{G(s) + sC(s)}}. \end{aligned}$$

For an uniform conductor, the S-parameter matrix is symmetrical [12]. Rearrange the representations of $S_{11}(s)$ and $S_{21}(s)$ in Equation (2.3), one has:

$$S_{11}(s) = \frac{[Z_c^2(s) - Z_0^2] \cdot \sinh(\gamma(s))}{2Z_0 Z_c(s) \cosh(\gamma(s)) + (Z_c^2(s) + Z_0^2) \sinh(\gamma(s))}, \quad (2.4)$$

$$S_{21}(s) = \frac{2Z_0}{2Z_0 Z_c(s) \cosh(\gamma(s)) + (Z_c^2(s) + Z_0^2) \sinh(\gamma(s))}. \quad (2.5)$$

In order to find the lower order approximations of $S_{11}(s)$ and $S_{21}(s)$, the representations of

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the $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$ and $\mathbf{G}(f)$ must first be found. A curve-fitting method is then adapted to find the coefficients of the polynomials which model the parasitics. At first, the linear fit is run on each data set to obtain two coefficients as the initial assignment for the successive curve-fitting steps. Then the problem of finding the coefficients of the polynomials are transformed into one of the least-square error estimation. The Levenberg-Marquardt method [20] is used to solve this least-square error estimation problem. This method is chosen because it combines the best features of both the Taylor series expansion and the gradient methods. It can find the best solution even it is outside of the circle of convergence like gradient methods and the rate of convergence is as fast as the Taylor series methods.

The two coefficients found in the linear fit is passed on to the Levenberg-Marquardt method as an initial guess. The Levenberg-Marquardt method then iterates to find the best fit coefficients for the curves of $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$ and $\mathbf{G}(f)$. This method stops when the results converges or the number of iteration exceeds a preset limit. For all of the experiments, this curve-fitting method shows better results than the one-pass least-square fit and the singular-value decomposition fit methods.

If one defines:

$$T(s) \equiv \gamma^2(s) \equiv \sum_{i=0}^q t_i \cdot s^i + o(s^q) \quad (2.6)$$

$$\begin{aligned} A(s) &\equiv Z_c(s)\gamma(s) - Z_0^2 \cdot \frac{\gamma(s)}{Z_c(s)} \\ &\equiv \sum_{i=0}^q a_i \cdot s^i + o(s^q) \end{aligned} \quad (2.7)$$

$$\begin{aligned} B(s) &\equiv Z_c(s)\gamma(s) + Z_0^2 \cdot \frac{\gamma(s)}{Z_c(s)} \\ &\equiv \sum_{i=0}^q b_i \cdot s^i + o(s^q), \end{aligned} \quad (2.8)$$

whereas $\gamma^2(s) = (R(s) + sL(s))(G(s) + sC(s))$, $\frac{\gamma(s)}{Z_c(s)} = G(s) + sC(s)$, and $Z_c(s)\gamma(s) = R(s) + sL(s)$, there is no square root involved in the evaluation of the approximations of the $T(s)$, $A(s)$, and $B(s)$ complex functions. The approximation of the $R(s)$, $L(s)$, $G(s)$, and $C(s)$ real functions are known through curve-fitting. The coefficients of the $T(s)$, $A(s)$, and $B(s)$ complex functions can be found through simple polynomial operations.

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If one further defines:

$$\begin{aligned} T(s) &\equiv t_0 + \sum_{i=1}^q t_i \cdot s^i + o(s^q) \\ &\equiv t_0 + T_q(s) + o(s^q). \end{aligned} \quad (2.9)$$

where the constant term is separated from the rest of function. The separation will make the finding of the coefficients in Equation (2.14) and (2.15) much easier. The expansions of $2Z_0 \cosh(\gamma(s))$ and $\frac{\sinh(\gamma(s))}{\gamma(s)}$ are shown in Equation (2.10) and (2.11):

$$\begin{aligned} U(s) &\equiv 2Z_0 \cosh(\gamma(s)) \equiv \sum_{i=0}^{\infty} \frac{\gamma^{2i}(s)}{2i!} \equiv 2Z_0 \sum_{i=0}^{\infty} \frac{T^i(s)}{2i!} \\ &\equiv 2Z_0 \sum_{i=0}^q u_i \cdot s^i + o(s^q) \end{aligned} \quad (2.10)$$

$$\begin{aligned} V(s) &\equiv \frac{\sinh(\gamma(s))}{\gamma(s)} \equiv \sum_{i=0}^{\infty} \frac{\gamma^{2i}(s)}{(2i+1)!} \equiv \sum_{i=0}^{\infty} \frac{T^i(s)}{(2i+1)!} \\ &\equiv \sum_{i=0}^q v_i \cdot s^i + o(s^q). \end{aligned} \quad (2.11)$$

Based on Equations (2.7), (2.8), (2.10), and (2.11), Equations (2.4) and (2.5) can be rewritten as:

$$S_{11}(s) = \frac{A(s) \cdot V(s)}{U(s) + B(s) \cdot V(s)} \quad (2.12)$$

$$S_{21}(s) = \frac{2Z_0}{U(s) + B(s) \cdot V(s)}. \quad (2.13)$$

It can be shown that:

$$U(s) = 2Z_0 \sum_{k=0}^q \beta_k \cdot T_q^k(s) + o(s^q) \quad (2.14)$$

$$V(s) = \sum_{k=0}^q \alpha_k \cdot T_q^k(s) + o(s^q), \quad (2.15)$$

where

$$\beta_k = \sum_{i=0}^{\infty} \frac{C_i^k}{2i!} \cdot t_0^{i-k} \quad (2.16)$$

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$$\alpha_k = \sum_{i=0}^{\infty} \frac{C_i^k}{(2i+1)!} \cdot t_0^{i-k}, \quad (2.17)$$

and t_0 is the separated constant term in Equation (2.9). Although the summation of both α_k and β_k are an infinity series, in reality, the inverse of the factorial is a fast converging series and they can be truncated at a certain point without introducing much error. The coefficients of $U(s)$ and $V(s)$ are:

$$u_i = \sum_{k=1}^i \beta_k \sum_{j_1+j_2+\dots+j_k=i} t_{j_1} \cdot t_{j_2} \cdots t_{j_k} \quad (2.18)$$

$$v_i = \sum_{k=1}^i \alpha_k \sum_{j_1+j_2+\dots+j_k=i} t_{j_1} \cdot t_{j_2} \cdots t_{j_k}. \quad (2.19)$$

Since the coefficients of the polynomial $T(s)$ are known from Equation (2.6), the coefficients of $U(s)$ and $V(s)$ can then be computed.

If one defines:

$$S_{11}(s) = \frac{C(s)}{D(s)} \quad (2.20)$$

$$S_{21}(s) = \frac{C'(s)}{D(s)}. \quad (2.21)$$

Comparing Equations (2.12), (2.13), (2.20), and (2.21), one finds:

$$C(s) \equiv \sum_{i=0}^q c_i \cdot s^i + o(s^q) = A(s) \cdot V(s) \quad (2.22)$$

$$\begin{aligned} D(s) &\equiv \sum_{i=0}^q d_i \cdot s^i + o(s^q) \\ &= U(s) + B(s) \cdot V(s) \end{aligned} \quad (2.23)$$

$$C'(s) = 2Z_0. \quad (2.24)$$

It can be shown that the coefficients of $C(s)$ and $D(s)$ are:

$$c_k = \sum_{i+j=k} a_i v_j \quad (2.25)$$

$$d_k = u_k + \sum_{i+j=k} b_i v_j. \quad (2.26)$$

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If the moments of $S_{11}(s)$ and $S_{21}(s)$ are m_i and n_i , one can write:

$$S_{11}(s) = \sum_{i=0}^q m_i \cdot s^i + o(s^q) \quad (2.27)$$

$$S_{21}(s) = \sum_{i=0}^q n_i \cdot s^i + o(s^q). \quad (2.28)$$

From Equation (2.20) and (2.28), one can derive:

$$\sum_{i+j=k} m_i d_j = c_k. \quad (2.29)$$

If one denotes:

$$D_q = \begin{bmatrix} d_0 & 0 & 0 & \dots & 0 \\ d_1 & d_0 & 0 & \dots & 0 \\ d_2 & d_1 & d_0 & \dots & 0 \\ \dots & & & & \\ d_q & d_{q-1} & d_{q-2} & \dots & d_0 \end{bmatrix} \quad (2.30)$$

$$M_q = \begin{bmatrix} m_0 & m_1 & m_2 & \cdot & \cdot & \cdot & m_q \end{bmatrix}^T \quad (2.31)$$

$$C_q = \begin{bmatrix} c_0 & c_1 & c_2 & \cdot & \cdot & \cdot & c_q \end{bmatrix}^T, \quad (2.32)$$

where T represents the transpose of the vector, one can rewrite $D_q M_q = C_q$, i.e. $M_q = D_q^{-1} C_q$.

Thus the moments m_i of the $S_{11}(s)$ function can be found through simple backward substitutions.

Similarly, if one denotes:

$$N_q = \begin{bmatrix} n_0 & n_1 & n_2 & \cdot & \cdot & \cdot & n_q \end{bmatrix}^T \quad (2.33)$$

$$C'_q = \begin{bmatrix} 2Z_0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix}^T, \quad (2.34)$$

where T again represents the transpose of the vector, one can rewrite $D_q N_q = C'_q$, i.e. $N_q = D_q^{-1} C'_q$, and the moments n_i of the $S_{21}(s)$ function can again be found through simple backward substitutions.

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material, the distance of the coupled lines which are in parallel, the rate of change of the input waveforms, as well as the line impedances.

Previous researches which used scattering parameter for the analysis of the coupled transmission lines include those of the general convolution by Winklestein et al. [26], the full-wave analysis by Cooke et al. [9] and the time domain transient simulation by Schutt-Aine et al. [24]. While Schutt-Aine et al. demonstrated a greater accuracy improvement in simulating the circuits that included non-linear drivers and terminations, their method involved costly matrix computation for converting the frequency domain scattering parameter matrix representation into the time domain transmission/reflection matrix representation [24]. Cooke et al. illustrated an ability to simulate frequency dependent modal propagation, but a time consuming full-wave analysis was required [9]. Recently, a novel frequency domain simulator using scattering parameter based macromodels has been presented by Liao et al [18] [19]. Based on the scattering parameter based macromodel, Pade technique or Exponentially Decayed Polynomial Function (EDPF) can be used to approximate transfer functions of the coupled interconnects. In the following section, we are going to try to overcome some of these shortcomings by deriving the S-parameter macromodel for the frequency-dependent coupled transmission lines.

Based upon the assumption of quasi-TEM wave propagation, the distributions of voltages and currents in a n coupled lossy transmission-line system can be described by the generalized Telegraphist's equations [7]:

$$\frac{\partial \mathbf{v}(x, t)}{\partial x} = -\mathbf{L}(\mathbf{f}) \frac{\partial \mathbf{i}(x, t)}{\partial t} - \mathbf{R}(\mathbf{f}) \mathbf{i}(x, t) \quad (2.35)$$

$$\frac{\partial \mathbf{i}(x, t)}{\partial x} = -\mathbf{C}(\mathbf{f}) \frac{\partial \mathbf{v}(x, t)}{\partial t} - \mathbf{G}(\mathbf{f}) \mathbf{v}(x, t), \quad (2.36)$$

where $0 \leq x \leq l$. $\mathbf{v}(x, t)$ and $\mathbf{i}(x, t)$ are column vectors defining the voltages distributions $v_k(x, t)$ and currents distributions $i_k(x, t)$ on the conductors $k = 1, 2, 3, \dots, n$. The $\mathbf{L}(\mathbf{f})$ and $\mathbf{C}(\mathbf{f})$ are the n by n symmetric matrices of the frequency-dependent per-unit-length inductance and capacitance of the n conductor system. The $\mathbf{R}(\mathbf{f}) = \text{diag}(R_{kk}(f))$, $k = 1 \dots n$ is the diagonal matrix of the frequency-dependent per-unit-length resistance of the n conductors. The $\mathbf{G}(\mathbf{f})$ is the n by n symmetric matrix of the frequency-dependent per-unit-length conductance of the n conductor system [24].

It is very important to accurately model the frequency dependence of the parasitics for the case

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of the coupled transmission lines. There are off-diagonal elements in the $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$ and $\mathbf{G}(f)$ matrices describing the mutual coupling effects which do not exist in the case of the single transmission line. The mutual inductance $\mathbf{L}_{ij}(f)$, where $i \neq j$, increases as frequency increases due to more coupling between lines at higher frequency. The mutual capacitance $\mathbf{C}_{ij}(f)$, where $i \neq j$, stays constant as the $\mathbf{C}_{ii}(f)$. Both the $\mathbf{R}_{ii}(f)$ and $\mathbf{R}_{ij}(f)$, where $i \neq j$, increase as the frequency rises.

There are two major methods to find the time domain transient response waveforms of a coupled transmission line system. One method is to find the impulse response of the linear coupled transmission line system and then use either the convolution or the waveform relaxation to find the time domain waveforms. However, this method suffers from both the large memory requirement and the long computation time that are required. The other method is a modal wave propagation decoupling method which is preferred over the first method because it models the physical phenomenon of n fundamental mode of the wave propagation that exists in the n multi-conductor transmission line system. By decoupling the modal waves, the simulator is only required to memorize a period of the waveforms equal to the time-of-flight of each decoupled transmission line, which is much shorter when compared to the duration of the impulse response. Furthermore, with the help of the S-parameter macromodel, the recursive convolution can be applied with a significantly shorter computation time. After successfully decoupling of the coupled transmission lines system, the computation of the scattering parameter macromodel of the entire system becomes the computation of the decoupling networks and that of the decoupled transmission lines with frequency-dependent losses. The macromodel of the later is already available and is presented in Section 2.1.

To incorporate the macromodel of the frequency-dependent decoupling networks into the S-parameter macromodel simulator presents a very difficult challenge. This process requires the finding of a frequency-dependent transformation matrix in order to decouple the system. It is a complex process and requires the eigenvalues at each frequency point prior to diagonalization. The resulting matrix elements are characterized by the tabulated S-parameter data.

Taking the Laplace transform of the Equation (2.35) and (2.36), they can be rewritten as:

$$\frac{\partial \mathbf{V}(x, s)}{\partial x} = -\mathbf{Z}\mathbf{I}(x, s) \quad (2.37)$$

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$$\frac{\partial \mathbf{I}(x, s)}{\partial x} = -\mathbf{Y}\mathbf{V}(x, s), \quad (2.38)$$

where $\mathbf{Z} = \mathbf{R} + jw\mathbf{L}$, and $\mathbf{Y} = \mathbf{G} + jw\mathbf{C}$. Throughout this paper, the following assumptions reported by Blazeck et al. are used [4]. The assumptions are: the modes of propagation must be TEM or quasi-TEM, and the lines are of uniform cross-section throughout their length; that is, \mathbf{R} , \mathbf{L} , \mathbf{C} , and \mathbf{G} are assumed to be constant with respect to the spatial variable x .

Solving Equation (2.37) and (2.38), one has:

$$\frac{\partial^2 \mathbf{V}(x, s)}{\partial x^2} = \mathbf{Z}\mathbf{Y}\mathbf{V}(x, s) = \mathbf{\Gamma}^2 \mathbf{V}(x, s) \quad (2.39)$$

$$\frac{\partial^2 \mathbf{I}(x, s)}{\partial x^2} = \mathbf{Y}\mathbf{Z}\mathbf{I}(x, s) = (\mathbf{\Gamma}^2)^T \mathbf{I}(x, s), \quad (2.40)$$

where T indicates transpose and $\mathbf{\Gamma}$ is defined as $\mathbf{\Gamma}^2 = \mathbf{Z}\mathbf{Y}$ [4]. The existence of eigenvectors that diagonalize $\mathbf{\Gamma}^2$, and thus $\mathbf{\Gamma}$, is also assumed throughout this paper [4]. Define

$$\mathbf{\Gamma} = \mathbf{X}\mathbf{\Lambda}\mathbf{X}^{-1}, \quad (2.41)$$

where \mathbf{X} is the eigenvectors of $\mathbf{\Gamma}$; therefore, they are also the eigenvectors of $\mathbf{\Gamma}^2$, and $\mathbf{\Lambda}$ is the diagonal matrix of the eigenvalues of $\mathbf{\Gamma}$. It can be shown that [25] [10]

$$\frac{\partial \mathbf{V}_{\mathbf{m}}(x, s)}{\partial x} = -\mathbf{\Lambda}\mathbf{Z}_{\mathbf{m}}\mathbf{I}_{\mathbf{m}}(x, s) \quad (2.42)$$

$$\frac{\partial \mathbf{I}_{\mathbf{m}}(x, s)}{\partial x} = -\mathbf{\Lambda}\mathbf{Y}_{\mathbf{m}}\mathbf{V}_{\mathbf{m}}(x, s), \quad (2.43)$$

where $\mathbf{V}(x, s) = \mathbf{X}\mathbf{V}_{\mathbf{m}}(x, s)$, $\mathbf{I}(x, s) = (\mathbf{X}^T)^{-1}\mathbf{I}_{\mathbf{m}}(x, s)$. The modal impedance matrix $\mathbf{Z}_{\mathbf{m}}$ and modal admittance matrix $\mathbf{Y}_{\mathbf{m}}$ are related to the eigenvector matrix \mathbf{X} and the impedance matrix \mathbf{Z} by

$$\mathbf{Z}_{\mathbf{m}} = (\mathbf{Y}_{\mathbf{m}})^{-1} = \mathbf{\Lambda}^{-1}\mathbf{X}\mathbf{Z}(\mathbf{X}^T)^{-1}. \quad (2.44)$$

With the eigenvector matrix \mathbf{X} , the original coupled transmission lines can be decoupled into two congruence transformer and a set of n decoupled transmission lines [7]. The task of finding the macromodel of the frequency-dependent coupled transmission lines becomes that of finding the

macromodel representations of the congruence transformers and the frequency-dependent single transmission lines [25]. The macromodel of the single transmission line that is characterized with frequency-dependent losses has already been developed in Section 2.1. The remaining task is to find the macromodel representation of a frequency-dependent congruence transformer.

Bayard first outlined the transformation, $\mathbf{A}^t \mathbf{Z} \mathbf{A}$, and called it “translator” [2]. Hazony is the first one to name the transformation “congruence transformer” in his book [15]. Chang used the congruence transformer to decouple both the lossless [5] and lossy coupled transmission lines [7]. Chang’s method for the analysis of coupled transmission lines relies on simultaneously diagonalizing all the matrices using a special conditioned matrix.

It is known that the modal eigenvectors of two symmetrical coupled transmission lines are frequency independent constant vectors even the lines are characterized by the frequency-dependent parasitics [13]. It can be shown that the decoupling networks can be constructed from the constant eigenvectors:

$$\mathbf{X} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}. \quad (2.45)$$

These two vectors correspond to the odd and even mode of propagation that exists in the symmetrical coupled transmission lines.

For asymmetrical coupled dual transmission lines and for coupled transmission lines with more than two conductors which are characterized with the frequency-dependent parasitic, the model structure of the lines becomes frequency-dependent [13], and finding the moments of the model structure will pose an even greater challenge.

It can be shown that for the coupled lossy transmission line systems, the congruence transformer matrix is the eigenvector matrix of the complex matrix $\mathbf{\Gamma}$ [4]. The existence of such eigenvector matrix \mathbf{X} that simultaneously diagonalize the complex matrices $\mathbf{\Gamma}$, $\mathbf{\Gamma}^2$, $\mathbf{Z}\mathbf{Y}$, and $\mathbf{Y}\mathbf{Z}$ is assumed throughout this paper. This assumption is also adopted by Gordon et al. [13], Blazeck et al. [4] and Schutt-Aine et al. [24].

Gordon et al. suggested that the frequency-dependent congruence transformer can be found by performing the congruence decoupling at each frequency point, and by checking the orthogonality of all the eigenvectors for all the frequency points [13]. If the eigenvectors were not orthogonal

to each other, column swapping must be performed so that for all the congruence transformation matrices at all the frequency points are orthogonal to any other one. During this process, the $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$ and $\mathbf{G}(f)$ matrices have been diagonalized to be: $diag(\mathbf{R}(f))$, $diag(\mathbf{L}(f))$, $diag(\mathbf{C}(f))$ and $diag(\mathbf{G}(f))$ eigenvalue matrices. The i -th diagonal eigenvalues of each frequency points constitute the parasitic of the decouple frequency-dependent single transmission line. Thus these diagonal eigenvalue matrices can be used to form the macromodels using a method outlined in the Section 2.1.

Once the tabulated S-parameter data for the congruence transformation have been found, a curve-fitting using Levenberg-Marquardt method [20] is used to find the coefficients for the construction of the congruence transformer: $X(f)$.

One has derived the scattering parameter matrix $S(s)$ of the congruence transformer $X(f)$ to be [25]:

$$S(s) = \begin{bmatrix} -[X^{-1}(f) + X^t(f)]^{-1}[X^{-1}(f) - X^t(f)] & 2[X(f) + (X^t(f))^{-1}]^{-1} \\ 2[X^{-1}(f) + X^t(f)]^{-1} & -[X(f) + (X^t(f))^{-1}]^{-1}[X(f) - (X^t(f))^{-1}] \end{bmatrix}, \quad (2.46)$$

where the sub-matrix $X(f)$ is found using curve-fitting the tabulated congruence transformation data.

Similar to the process stated in an earlier paper [25], the macromodels of the two congruence transformers and the n decoupled transmission lines are passed onto the S-parameter macromodel based simulator to perform the transient analysis.

3 Experimental Results

The data in the first two examples as well as the fourth example presented here are obtained from Dr. J. C. Liao of Intel Corporation. The data in the third example is obtained from the user manual of Dr. Raji Mittra's "mtltda" simulator. Figure 3.1 (b) only shows the simulation result which takes frequency-dependent losses into consideration. Figure 3.2 shows the different simulation results between *taking* and *not taking* the frequency-dependent losses into consideration. The discrepancy in simulation waveforms confirms that one needs to include frequency-dependent losses in circuit simulation. Figure 3.3 and Figure 3.4 show the comparison between the S-parameter macromodel based simulator and a time-domain circuit simulator. Although the S-parameter macromodel based

simulator does not have the accuracy demonstrated by the time-domain simulator, it provides more than thirty times speedup in Example 4. This kind of efficiency lands it in the application of performance-driven layout synthesis.

3.1 Example 1

This is a single transmission line characterized with frequency-dependent losses. The frequency dependence of the per-unit-length inductance and resistance are given in Table 3.1. The per-unit-

Frequency	L (nH/cm)	R (ohm/cm)
10 kHz	4.070	5.000
100 MHz	4.069	5.000
250 MHz	4.064	5.000
500 MHz	4.050	5.150
750 MHz	4.032	5.310
1 GHz	4.012	5.520
2 GHz	3.904	6.750
4 GHz	3.789	8.960
6 GHz	3.724	10.85
8 GHz	3.645	12.35

Table 3.1: **The Frequency-Dependent Per-Unit-Length Inductance and Resistance.**

length capacitance is $1.460pF/cm$ and the per-unit-length conductance is assumed to be zero. The driving signal is 100-MHz, 50% duty-cycle pulse with $0.5ns$ rise/fall time. The Far-end waveforms are simulated with time-of-flight extracted [16]. The circuit schematic is shown in 3.1 (a) with the component values. The simulation waveforms of this example are shown in Figure 3.1 (b).

3.2 Example 2

This is an example with two coupled transmission lines characterized with frequency-dependent losses. The frequency-dependent per-unit-length inductance and resistance are given in Table 3.2. The per-unit-length capacitance matrix is a constant matrix which does not vary with frequency:

$$\begin{bmatrix} 1.637pF/cm & -0.177pF/cm \\ -0.177pF/cm & 1.637pF/cm \end{bmatrix} \quad (3.1)$$

- using the frequency-independent macromodel.
- using the frequency-dependent macromodel.

The per-unit-length parasitic of the lossy macromodel are taken from the DC values of the frequency-dependent model. The frequency-independent macromodel is created using the DC values of the frequency-dependent model at all the frequency points. The results show that the output waveforms of the frequency-independent macromodel match that of the lossy macromodel as expected. The results also indicate that the output waveforms of the frequency-dependent macromodel differ from that of the lossy and frequency-independent model because of the frequency-dependent natural of the per-unit-length parasitic. This prompts the importance of taking frequency-dependent losses into consideration when doing circuit simulation.

3.3 Example 3

This is an example of two cascade sections of two coupled transmission lines, one section is characterized with frequency-dependent losses, and the other is characterized only with lossless model. The frequency-dependent per-unit-length inductance and resistance of the first section are given in Table 3.3. The frequency-dependent per-unit-length capacitance matrix of the first section

Frequency	L_{11} L_{22} (nH/cm)	L_{12} L_{21} (nH/cm)	R_{11} R_{22} (ohm/cm)	R_{12} R_{21} (ohm/cm)
DC	5.150	0.995	0.6406	0.1283
20 MHz	4.777	0.943	0.6829	0.1325
50 MHz	4.511	0.818	0.7793	0.1736
100 MHz	4.354	0.818	1.0147	0.2359
200 MHz	4.022	0.688	1.2572	0.2609
500 MHz	3.601	0.588	2.1743	0.4284
1 GHz	3.409	0.580	3.6886	0.6957

Table 3.3: **The Frequency-Dependent Per-Unit-Length Inductance, Mutual Inductance, and Resistance.**

is given in Table 3.4. The per-unit-length conductance is assumed to be zero.

The second section is characterized as a lossless coupled transmission lines. The per-unit-length inductance and capacitance matrices are as follows:

$$\mathbf{L} = \begin{bmatrix} 5.105nH/cm & -0.995nH/cm \\ -0.995nH/cm & 5.105nH/cm \end{bmatrix}$$

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Frequency	$C_{11} C_{22}$ (pF/cm)	$C_{12} C_{21}$ (pF/cm)
DC	0.862	-0.140
20 MHz	0.752	-0.159
50 MHz	0.756	-0.140
100 MHz	0.754	-0.153
200 MHz	0.786	-0.147
500 MHz	0.768	-0.142
1 GHz	0.766	-0.146

Table 3.4: The Frequency-Dependent Per-Unit-Length Capacitance Matrix.

$$\mathbf{C} = \begin{bmatrix} 0.862pF/cm & -0.140pF/cm \\ -0.140pF/cm & 0.862pF/cm \end{bmatrix} \quad (3.2)$$

The driving signal is 100-MHz, 50% duty-cycle pulse with 0.5ns rise/fall time. The Far-end waveforms are simulated with time-of-flight extracted [16]. The circuit schematic is shown in 3.3 (a) with the component values. The simulation waveforms of this example are shown in Figure 3.3 (b), (c), (d) and (e).

For all of the figures, there are two output waveforms, they are:

- the output waveform of the frequency-dependent macromodel.
- the output waveform obtained from Dr. Mittra's "mtltda" simulator.

The results show that the output waveforms of the frequency-dependent macromodel match the one produced by Dr. Mittra's "mtltda" simulator as expected. The seemingly differences that were shown on the sense line waveforms are due to the lower number of order chosen and the Pade approximation error. Please note, however, the amplitude of the waveforms on the sense line are only one to two hundred millivolts. These waveforms, although different in large, have the same amplitude magnitude and tracking each other closely. Both simulators use 12 seconds on a SUN Sparc1+ workstation. However, the second data point for $\mathbf{R}(\mathbf{f})$ and $\mathbf{L}(\mathbf{f})$ is taken at 20MHz. As evident in the data that includes skin effects, there are some information lost in this simplification. If one includes these informations, the time-domain simulator will needs much longer simulation time.

3.4 Example 4

This is an example with three coupled transmission lines characterized with frequency-dependent losses. The per-unit-length inductance, resistance, and capacitance are not given in order to conserve space. The per-unit-length conductance is assumed to be zero. The driving signal is 200-MHz, 50% duty-cycle pulse with $0.5ns$ rise/fall time. The Far-end waveforms are simulated with time-of-flight extracted [16]. The circuit schematic is shown in 3.4 (a) with the component values. The simulation waveforms of this example are shown in Figure 3.4 (b), (c), (d) and (e).

Similar to Example 3, for all of the figures, there are two output waveforms, they are:

- the output waveform of the frequency-dependent macromodel.
- the output waveform obtained from Dr. Mittra's "mtltda" simulator.

Again, all the results show that the output waveforms of the frequency-dependent macromodel match the one produced by Dr. Mittra's "mtltda" simulator as expected. The seemingly differences that were shown on the sense line waveforms are due to the lower number of order chosen and the Pade approximation error. In this particular example, increase the order of Pade approximation does not increase accuracy, one must turn to other method such as Complex-Frequency-Hopping (CFH) or Pade-via-Lancsoz (PVL) to solve this accuracy problem.

The S-parameter macromodel based simulator takes 9.23 seconds on the SUN Sparc1+ workstation. The time-domain simulators takes 288 seconds on the same machine. This is due to the fact that the second data point is taking at $1MHz$ instead at $20MHz$. In order to use the time-domain simulator, the actual data taking at $10KHz$ and $100KHz$ are thrown away.

4 Conclusions

The task of designing interconnect networks for today's high performance digital systems requires an accurate and a more efficient transient analysis which takes the frequency-dependent losses into consideration. The contribution of this paper is to develop the two novel macromodels for both the single and coupled transmission lines characterized with the frequency-dependent parasitic functions $\mathbf{R}(f)$, $\mathbf{L}(f)$, $\mathbf{C}(f)$, and $\mathbf{G}(f)$ data in order to perform an accurate and a more efficient transient simulation. These two novel macromodels provides a moderate accuracy and a

huge speedup in simulation. This trade-off between accuracy and efficiency has to be made in order to play what-if scenario for a performance-driven layout synthesis which thousands of simulations must be executed to obtain timing and amplitude information. The S-parameter macromodel simulator with enough accuracy yet does not take a long time to simulate is well suitable for this kind of layout synthesis application.

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