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Figure 11. The result comparison of the lossy line circuit by SPICE3e2 and the 4th order approximation with **non**-time-of-flight (nonTOF)

Figure 12. The comparison of the lossy line circuit by SPICE3e2 and the 4th order approximation with time-of-flight (TOF).

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Figure 9. The result comparison of the clock network by SPICE3e2 and the 4th order approximation with time-of-flight (TOF)

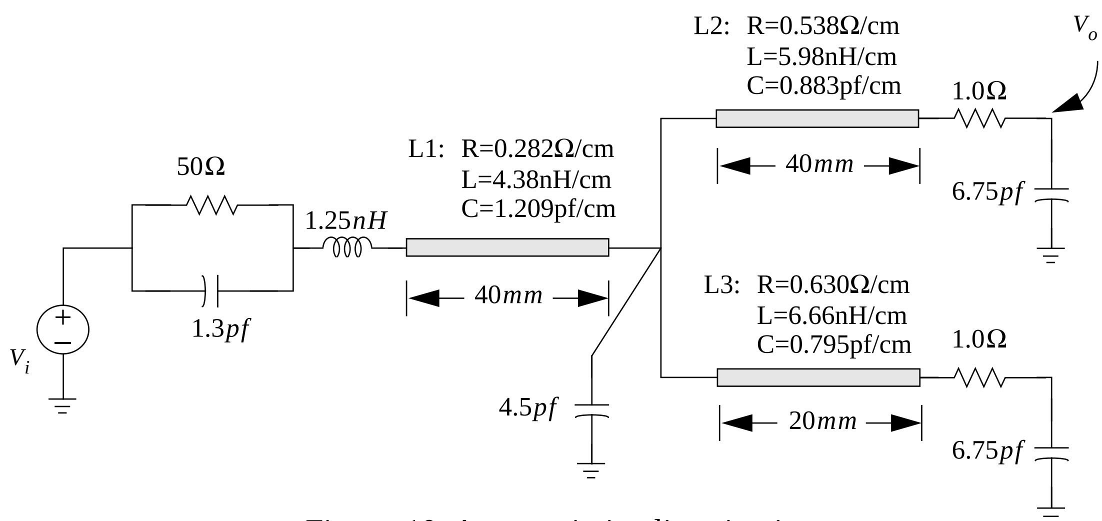

Figure 10. A transmission line circuit.

flight can be accurately extracted for computing the propagation delay. The accuracy of approximation of output responses, due to the extraction of the time-of-flight, can be greatly improved.

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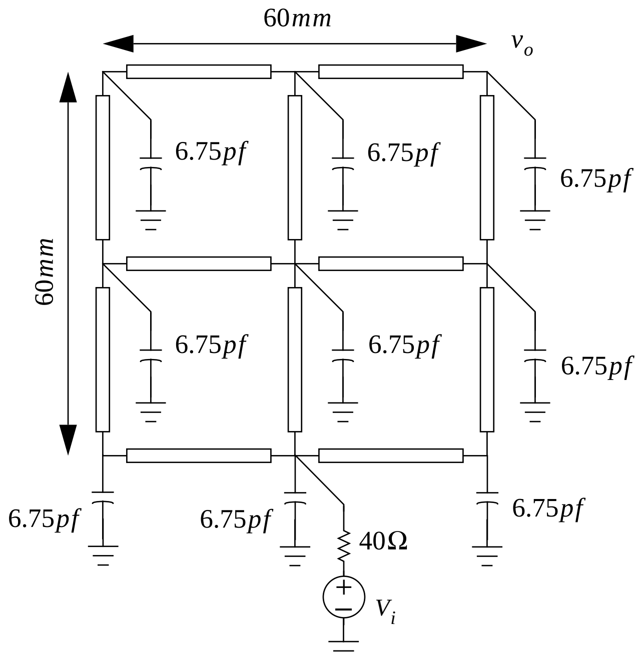

Figure 7. Grid-type clock network.

Figure 8. The result comparison of the clock network by SPICE3e2 and the 4th order approximation with **non**-time-of-flight (nonTOF)

5 Conclusions

A novel and efficient method for modeling arbitrary interconnect systems for transient simulation was presented. The method is based on scattering parameter technique, a large scale interconnect system can be reduced to a network containing one multiport component (macromodel) together with sources, loads of interest. While computing lower order approximation of the macromodel for evaluating the exponentially charging time, the time-ofmanipulation keeping track of time-of-flight explicitly, we can get the transfer function with the following form:

$$H(s) = e^{-\tau_s s} H'(s) \tag{24}$$

where $H'(s)$ is in Taylor series form with the term at $s = \infty$. A mixed exponential function $h'(t)$ [10] which combines Pade technique [12] and EDPF technique [4, 9] is used to match $H'(s)$. The corresponding $h'(t)$ has the form

$$h'(t) = \sum_{i=1}^{n_p} k_i e^{p_i t} + \sum_{j=0}^{n_e} c_i \left(\frac{t}{d}\right) e^{-t/d} \tag{25}$$

That is, $h'(t)$ is a sum of n_p exponentials and an n_e order exponentially decayed polynomial function. In Eq. (25), p_i and k_i are poles and residues of the exponentials, d and c_j are the time constant and coefficients of the exponentially decayed polynomial function respectively. The mixed exponential function is always stable for stable systems and have higher accuracy[10]. Thus the corresponding inverse Laplace transform of $H(s)$ is

$$h(t) = h'(t - \tau_f) u(t - \tau_f) \tag{26}$$

4 Experimental Results

Several testing circuits are used to verify the efficiency and generality of the macromodel with the extraction of the time-of-flight. These testing circuits include various topologies commonly encountered in the delay modeling of VLSI/MCM interconnects. All benchmarks were executed on a Sun Sparc 1+ station.

The first example is a grid-type clock network (See Figure 7) which is distributed around the periphery of a $60mm \times 60mm$ chip. The vertical runs are on metal 1 ($R = 0.04\Omega/mm$, $L = 0.5025nH/mm$ and $C = 0.1552pF/mm$) and the horizontal runs are on metal 2 ($R = 0.06\Omega/mm$, $L = 0.548nH/mm$ and $C = 0.1423pF/mm$). The network is represented by distributed lossy transmission lines. Figure 8 is the analysis result of SPICE3e2 and the 4th order approximation without extracting the time-of-flight. Figure 9 is the result of SPICE3e2 and the 4th order approximation, extracting the time-of-flight $(0.795ns)$. While using the same order approximation, the simulator with extraction of time-of-flight has much higher accuracy, since it only need to match the exponentially charging section of the response curves.

Figure 10 is a lossy transmission line circuit. Figure 11 is the analysis results with SPICE3e2 and the 4th order approximation without extracting the time-of-flight. Figure 12 is the result of SPICE3e2 and the 4th order approximation, extracting the time-of-flight $(0.582ns)$. The accuracy of response curves, not only the flat section, but whole curves has been greatly improved.

The four basic operations Eq. (13 - 16) together with the S-matrix of lossy transmission lines provide a novel method to accurately compute the time-of-flight of a general interconnect system. However, computing the S-matrix of the reduced network for every frequency point is time consuming. In order to speed up the reduction process, we expand the S-parameters of components by Taylor series, and derive the formula for manipulating two Taylor series, which are the extension of the work reported in [9].

Let

$$A'(s) = \left(A'(\infty), \sum_{i=0}^{n} a_i s^i\right) \text{ and } B'(s) = \left(B'(\infty), \sum_{i=0}^{n} b_i s^i\right) \tag{17}$$

where terms at $s = \infty$ are used to match the initial condition of transfer functions in time domain. The series addition/subtraction

$$C'(s) = A'(s) \pm B'(s) = \left(C'(\infty), \sum_{i=0}^{n} c_i s^i\right) \text{ where } C'(\infty) = A'(\infty) \pm B'(\infty) \text{ and } c_i = a_i \pm b_i \tag{18}$$

The series multiplication

$$D'(s) = A'(s)B'(s) = \left(D'(\infty), \sum_{i=0}^{n} d_i s^i\right) \text{ where } D'(\infty) = A'(\infty)B'(\infty) \text{ and } d_i = \sum_{j=0}^{i} a_j b_{i-j} \tag{19}$$

And the series division

$$E'(s) = \frac{A'(s)}{B'(s)} = \left(E'(\infty), \sum_{i=0}^{n} e_i s^i\right) \text{ where } E'(\infty) = \frac{A'(\infty)}{B'(\infty)} \text{ and } e_i = \frac{1}{b_0}\left(a_i - \sum_{j=0}^{i-1} d_j b_{i-j}\right) \tag{20}$$

The denominators will not be zero, since the scattering parameters of passive components have no poles at $s = 0$ or $s = \infty$, based on the principle of energy conservation[5].

The scattering parameters defined in Eq. (2, 3, 4 and 6) can be easily expanded into Taylor series based on Eq. (18 - 20). The S-parameters of lossy transmission lines (See Eq. (6 - 8)) at $s = \infty$ becomes:

$$S'_{11}(\infty) = S'_{22}(\infty) = \frac{\sqrt{L} - \sqrt{C}Z_0}{\sqrt{L} + \sqrt{C}Z_0} \tag{21}$$

$$\text{and } S'_{12}(\infty) = S'_{21}(\infty) = \begin{cases} \dfrac{4\sqrt{LC}Z_0}{(\sqrt{L} + \sqrt{C}Z_0)^2} & R = 0 \\ 0 & R \neq 0 \end{cases} \tag{22}$$

The exponential factor in Eq. (14) can be expanded into Taylor series based on the formula:

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots \tag{23}$$

Based on S-matrix with time-of-flight extracted and Taylor series expansion and

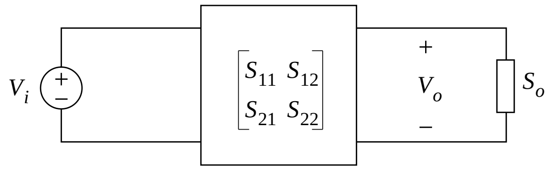

Figure 6. A two port network.

outgoing wave at port j to the incoming wave at port i. If both port i and port j belong to the same component, say X, then S_{ji} consists of two terms, in the other word, there are two paths for electromagnetic wave to propagate from port i to port j. The first term of Eq. (9a) stands for the first path on which the wave directly propagates from port i to port j. The second term for the second path on which the wave propagates from port i to port k, then reflected to port j. Obviously, the time-of-flight of the S_{ji} is equal to the less one of these two paths, and the time-of-flight of the second path is the sum of those of $S_{ki}^{(X)}$ and $S_{jk}^{(X)}$.

Similarly, if port i belongs to component X and port j belongs to component Y, then there is only one path for electromagnetic wave to propagate from port i to port j. The Eq. (9b) shows that wave propagates from port i to port k of component X, then from port l to port j of component Y. Note port k of component X and port l of component Y are connected together. The same analysis methods could be applied to Eq. (10) for self merging and Eq. (12) for transfer function.

In order to keep track of the time-of-flight delay, we derived the following basic operations:

Let $A(s) = e^{-\tau_A s}A'(s)$, $B(s) = e^{-\tau_B s}B'(s)$, then the addition/subtraction is defined as

$$C(s) = A(s) \pm B(s) = e^{-\tau_C s}C'(s) \tag{13}$$

where $\tau_C = min(\tau_A, \tau_B)$, and

$$C'(s) = \begin{cases} e^{-(\tau_A - \tau_B)s}A'(s) \pm B'(s) & (\tau_A \geq \tau_B) \\ A'(s) \pm e^{-(\tau_B - \tau_A)s}B'(s) & (\tau_A < \tau_B) \end{cases} \tag{14}$$

The multiplication is

$$D(s) = A(s)B(s) = e^{-\tau_D s}D'(s) \tag{15}$$

where $\tau_D = \tau_A + \tau_B$, and $D'(s) = A'(s)B'(s)$. The division is defined as

$$E(s) = A(s)/B(s) = e^{-\tau_E s}E'(s) \tag{16}$$

where $\tau_E = \tau_A$, $E'(s) = A'(s)/B'(s)$. Here we assume $\tau_B = 0$ for division, since the time-of-flights of the denominators in Eq. (9, 10 and 12) are all equal to zero due to the constant one in the denominators whose time-of-flight is zero.

merging X and Y, the resultant $(n+m-2)$ port has the following s-parameters:

$$S_{ji} = \begin{cases} S_{ji}^{(X)} + \dfrac{S_{ki}^{(X)} S_{ll}^{(Y)} S_{jk}^{(X)}}{1 - S_{kk}^{(X)} S_{ll}^{(Y)}} & i, j \in X \\ \dfrac{S_{ki}^{(X)} S_{jl}^{(Y)}}{1 - S_{kk}^{(X)} S_{ll}^{(Y)}} & i \in X, j \in Y \end{cases} \tag{9}$$

Self Merging Rule: Let X be an m port with a self loop connected to the l^{th} and k^{th} ports, as shown in Figure 3. After eliminating the self loop, the resultant (m - 2) port has the

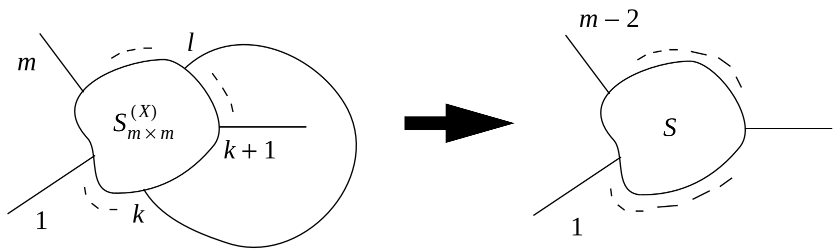

Figure 5. Self Merging.

following s-parameters:

$$S_{ji} = S_{ji}^{(X)} + S_{jl}^{(X)} a_l + S_{jk}^{(X)} a_k \qquad i, j = 1, 2, \ldots, m-2 \tag{10}$$

where

$$\begin{aligned} a_l &= \frac{1}{\Delta} (S_{li}^{(X)} S_{kk}^{(X)} + S_{ki}^{(X)} (1 - S_{lk}^{(X)})) \\ a_k &= \frac{1}{\Delta} (S_{ki}^{(X)} S_{ll}^{(X)} + S_{li}^{(X)} (1 - S_{kl}^{(X)})) \\ \Delta &= (1 - S_{lk}^{(X)})(1 - S_{kl}^{(X)}) - S_{ll}^{(X)} S_{kk}^{(X)} \end{aligned} \tag{11}$$

For an arbitrary distributed-lumped network described by the linear components, the Adjoined Merging Rule is used to merge all internal components, and the Self Merging rule is applied to eliminate the self loops introduced by the Adjoined Merging process. The macromodel, or the voltage transfer function of the network can be characterized by the s-parameters of the multiport component resulted from the reduction process, together with the s-parameters of the loads. From this reduced network, we can easily get the transfer function. For example, the voltage transfer function of the network shown in Figure 6 is

$$H(s) = \frac{S_{21}(1 + S_o)}{1 + S_{11} + S_o (S_{12} S_{21} - S_{11} S_{22} - S_{22})} \tag{12}$$

where S_o is the s-parameter of the load.

Time-of-flight is the shortest time that the output has the response after the input excites a signal. In order to extract the time-of-flight, let us go back the Eq. (9). S_{ji} relates the lossy transmission line, we find that the time-of-flights of both S_{11} and S_{22} are zero, but the time-of-flights of S_{12} and S_{21} are $\tau_f = \sqrt{LC}l$. Let us rewrite the Eq. (5),

$$S = \begin{bmatrix} S'_{11} & e^{-\tau_f s} S'_{12} \\ e^{-\tau_f s} S'_{21} & S'_{22} \end{bmatrix} \tag{6}$$

where

$$S'_{11} = S'_{22} = \frac{(Z_c^2 - Z_0^2)\sinh(\gamma)}{2Z_0 Z_c \cosh(\gamma) + (Z_c^2 + Z_0^2)\sinh(\gamma)} \tag{7}$$

$$S'_{12} = S'_{21} = \frac{2Z_c Z_0 e^{\tau_f s}}{2Z_0 Z_c \cosh(\gamma) + (Z_c^2 + Z_0^2)\sinh(\gamma)} \tag{8}$$

3 Network Reduction Keeping Track of Time-of-flight

Given the individual component scattering parameters, we have described a systematic reduction algorithm [8, 9] to reduce a distributed-lumped network to a multiport with sources and loads of interest, as shown in Figure 3. These reduction are based on two basic rules:

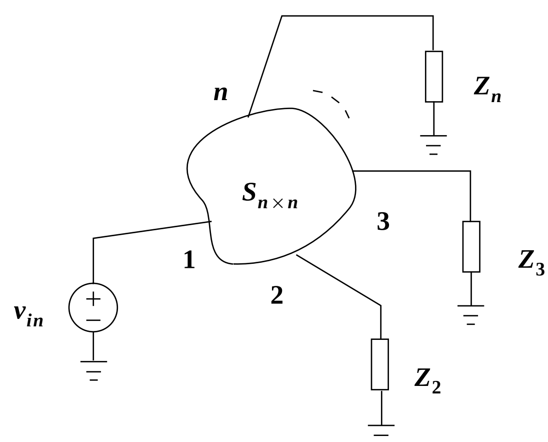

Figure 3. S-parameter based macromodel

Adjoined Merging Rule: Let *X* and *Y* be two adjacent multiports, with *m* ports and *n* ports respectively. Assume port *k* of *X* is connected to port *l* of *Y*, as shown in Figure 4. After

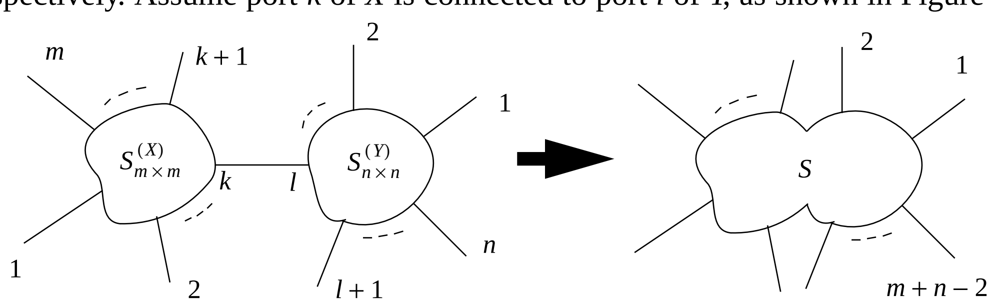

Figure 4. Adjoined Merging

multiport interconnect node and 4) lossy transmission line (See Figure 2).

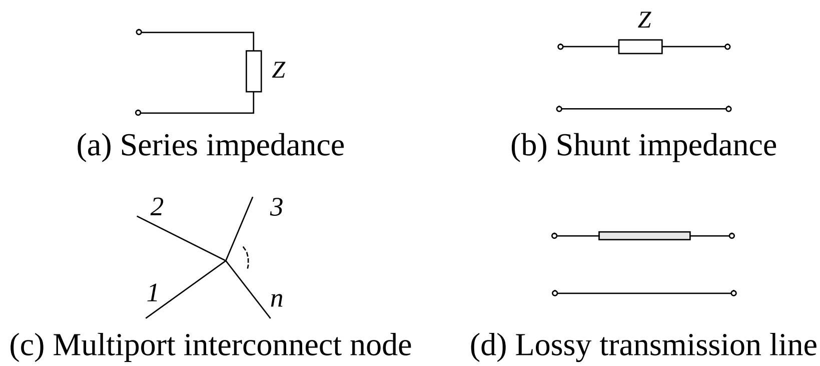

Figure 2. Four basic elements

For a one-port component with the impedance Z shown in Figure 2(a), its S-parameter is expressed as:

$$S = \frac{Z - Z_0}{Z + Z_0} \tag{2}$$

where Z_0 is an arbitrary reference impedance. The two-port component has the following S-matrix:

$$S = \frac{1}{Z + 2Z_0}\begin{bmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{bmatrix} \tag{3}$$

The multiport interconnect node as shown in Figure 2(c) can be used to connect all other components to construct a general interconnection system. The n-port interconnect node can be described by the scattering matrix [7]:

$$S = \frac{1}{n}\begin{bmatrix} 2-n & 2 & \dots & 2 \\ 2 & 2-n & \dots & 2 \\ \dots & \dots & \dots & \dots \\ 2 & 2 & \dots & 2-n \end{bmatrix} \tag{4}$$

Three components above are all lumped components, i.e., electromagnetic waves propagate across the component virtually instantaneously. Therefore, the time-of-flights of S-parameters described in Eq. (2-4) are all equal to zero.

For an RLC transmission line shown in Figure 2(d), we have

$$S = \frac{1}{2Z_0Z_c\cosh(\gamma) + (Z_c^2 + Z_0^2)\sinh(\gamma)} \times \begin{bmatrix} (Z_c^2 - Z_0^2)\sinh(\gamma) & 2Z_cZ_0 \\ 2Z_cZ_0 & (Z_c^2 - Z_0^2)\sinh(\gamma) \end{bmatrix} \tag{5}$$

where $Z_c = \sqrt{(R + sL)/(sC)}$ is the characteristic impedance, $\gamma = \sqrt{(R + sL)sC}l$ the wave propagation constant and l the length of the line. Applying Eq. (1) to the S-matrix of the

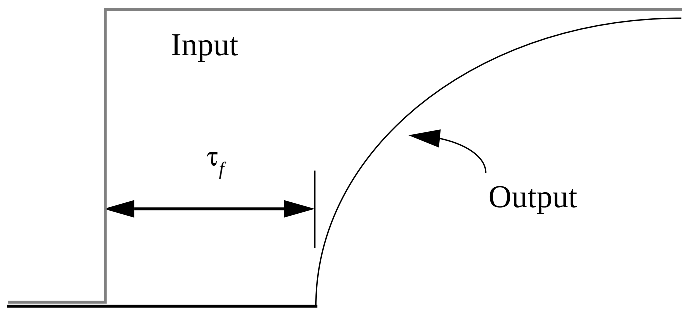

Figure 1. Time-of-Flight.

network will be $e(t-\tau_f)$. The response is the same as the excitation except that it is delayed in time by an amount τ_f. Therefore, the time-of-flight for an arbitrary transfer function $H(s)$ is defined [1] as

$$\tau_f = \lim_{s \to \infty} -\frac{1}{2}\frac{d}{ds}\left[ln\frac{H(s)}{H(-s)}\right] \tag{1}$$

While finding the explicit analytical expression of the transfer function for an arbitrary interconnect system to compute the time-of-flight is impractical, several attempts have been made to extracted the time-of-flight delay. They either require an explicit analytical expression of the transfer function[3] or can only deal with one set of transmission lines[11]. The extracted time-of-flight of one transmission line is also used as the lower bound of the delay for the lossy transmission lines[13].

This paper presents a new method to compute the time-of-flight for arbitrary interconnect systems, not limited to one transmission line. The method is based on a scattering parameter macromodel [8, 9]. First, we extract the time-of-flight of scattering parameter for basic components, then an effective network reduction is developed to compute the lower order macromodel of an interconnect system, keeping track of time-of-flight delay. The output responses, due to the extraction of the time-of-flight, is greatly improved.

2 S-parameters of Components with Time-of-Flight extracted

We use scattering parameters (S-parameters) to describe the components of interconnect systems. A scattering matrix is employed to relate outgoing waves to incoming waves of a multiport [5]. In order to extract time-of-flight of interconnect systems, let us first review S-parameters of some basic components of an interconnect system.

The components utilized to characterize a general interconnect network can be classified into four types [9]: 1) series (one port) impedance, 2) shunt (two port) impedance, 3)

Extracting Time-of-Flight Delay from Scattering Parameter Based Macromodel

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1 Introduction

As the electrical length of interconnects becomes a significant fraction of signal wavelength during the fast transient, the conventional lumped-impedance interconnect model becomes inadequate and transmission line effects must be taken into account for both on-chip and off-chip interconnects. The Elmore delay[6] based estimation methods, although efficient, are insufficient and the deficiencies of such methods need to be addressed with techniques which are capable of computing delays of a network containing RLC meshes, capacitive cutsets, inductive loops, and lossy transmission lines.

Recently, an n^{th} order extension of Elmore delay model based on Pade approximation has been developed [2, 11] to approximate a higher order linear network using the waveforms generated by its lower order moments. However, the delay associated with transmission line networks consists of the exponentially charging time and a pure propagation delay representing the finite propagating speed of electromagnetic signals in the dielectric medium. This propagation delay, so called time-of-flight delay, denoted by τ_f, is particularly evident in long lines (See Fig. 1). As time-of-flight of the signal across the interconnect is greater than, or comparable to, the input signal rise-time (i.e. long interconnects), it is impossible to model the perfectly flat response for $t \le \tau_f$ by a finite order of approximation[13], whether a finite sum of exponentials[2, 11] or an exponentially decayed polynomial function [4, 9].

Hence, the time-of-flight τ_f, more precisely the factor $e^{-s\tau_f}$, must be extracted from the transfer function of the circuit. As we know, a transfer function will be called ideal if it is of the form $H(s) = e^{-s\tau_f}$. For $s = j\omega$, the magnitude identically equals to one, and the angle is proportional to the angle frequency ω. According to the shifting theorem of Laplace transform, if this ideal network is excited by a signal $e(t)$, the corresponding response of the

Extracting Time-of-Flight Delay from Scattering Parameter Based Macromodel

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ABSTRACT

A novel and efficient method for transient analysis of interconnect systems is presented. The method is based on the scattering parameter technique. By extracting the propagation delay of the lossy transmission line and developing an efficient network reduction method, we compute the lower order model of the interconnect system for evaluating the exponentially charging time, keeping track of time-of-flight delay. The accuracy of the system response can be greatly improved from the extraction of the time-of-flight.

Keywords: Time-of-flight, propagation delay, scattering parameter, macromodel, multiport component, multiport interconnect node, component merging, network reduction, lossy transmission line