Force-Driven
Constrained Wiring Optimization

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Abstract

We describe a practical iterative approximation algorithm that can be used for homotopic positioning of Steiner points and vias for minimization of a wiring cost in rubber-band sketches. The algorithm is based on modeling the interconnect as a physical system and iterativly finding it minimum energy state. This method has a wide range of applications including finding Steiner trees, optimization of nets with given topologies, and optimization of constrained nets. It supports multi-layer sketches with obstacles, and can be adapted to various optimization goals by changing the force function. The algorithm can be used to improved the topology or to improve the wiring for a given topology. By controlling, the force functions of the physical system, various aspect of the interconnect such as average or maximal wire length can be improved.

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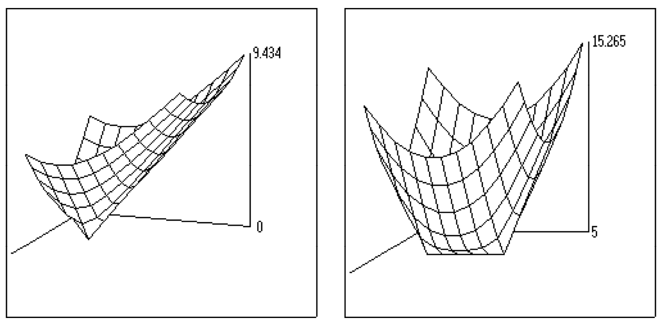
1. The Basic Case - Single Steiner Point

This problem, defined by Weber, illustrates our approach on a relatively simple case. Let $P_i=(x_i,y_i)$, $i=1..n$, $n>0$ be a set of non movable points on the plane. The goal is to find the position of a point $S=(x,y)$ such that the sum of the Euclidean distances from S to P_i is minimized. Figure 2a shows a problem instance with $n=3$, and with a non optimal solution S.

Let $L_i(x,y)$ be the distance from P_i to the moveable point $S=(x,y)$ and let $L(x,y)$ be the sum of $L_i(x,y)$. $L(x,y)$ and $L_i(x,y)$ form surfaces over the plane where Z(x,y) is the wiring length when the movable point is at (x,y). Figure 1a shows a L_i surface and a L surface (with two points, $n=2$). The optimal solutions to the problem are exactly all the global minimums of L.

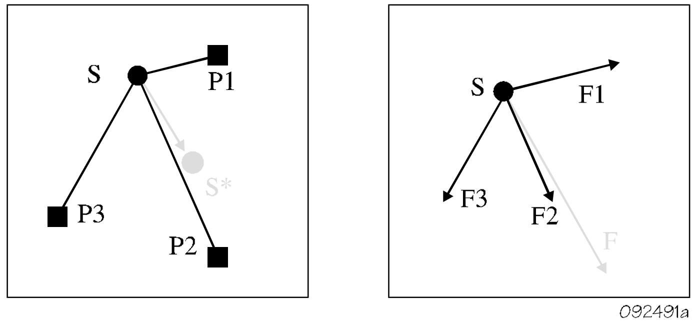


(a) (b)

Figure 1 - The surfaces of a single (left) and a pair (right) of non movable points. For a single point, the surface is a cone. For multiple points, the surface is sum of cones.

The solution to this problem can be approximated to any required accuracy using numeric analysis techniques. An initial point is selected and then it is improved iteratively by seeking a global minimum. We have shown that for those surfaces, every local minimum is also a global minimum, and for every two local (and global) minimum points, there is a path between them which includes only local minimum points (the proof appears latter in this paper). These properties support a greedy approach which at every step, tries to improve the most. Since the surface does not have local minimum which are not global minimum, the greedy approach can not be trapped in a local minimum.

We model the problem as a physical system of fixed force springs. The n wire segments which connect the movable point S to the non-movable points Pi, apply equal forces on S, each in its own direction. The combined force V of the n forces (Figure 2b) has a direction and magnitude, and represent the combined force applied on S. We have shown (the proof appears latter in this paper) that the direction of this force is the direction where the slope of the surface is the highest so maximum improvement is achieved by a small movement of S in that direction. In addition, the magnitude of the combined force is proportional to the improvement achieved by moving S in this direction.



(a) (b)

Fig 2 - An improvement step. The tree on the left side has three fixed points P1, P2, P3, and one moving point S. The vectors on the right are the forces operating on point S. The combined force V defines a movement vector which relocates S to S*.

We have implemented such a greedy algorithm which at every iteration relocates S in the direction of combined force, and in a distance proportional to the magnitude of the combined force. The distance of the movement of *S* is the magnitude of the combined force multiplied by a constant *C*. To get closer and closer approximation of the optimal solution, we decrease *C* gradually so the steps are finer and finer. We have tested the algorithm with a wide range of examples and in all cases the algorithm converged to an optimal solution.

2. Steiner Tree of Fixed Topology

The same iterative method can be extended to solve continuos, Euclidian, Steiner tree problems for a given topology. This is the case when the wiring is done according to predefined topologies and wiring rules. At every iteration, the new location of all the Steiner points are calculated, and then, they are moved to those locations. Another possible approach is at every step, to move only the point that has the highest force and thus, highest improvement is expected.

Preliminary experiments have shown that the this greedy algorithm converges quickly and consistently to the optimal solution.

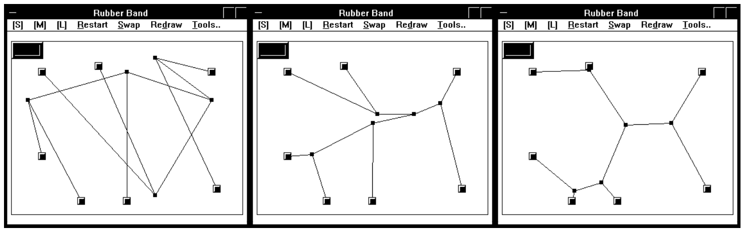
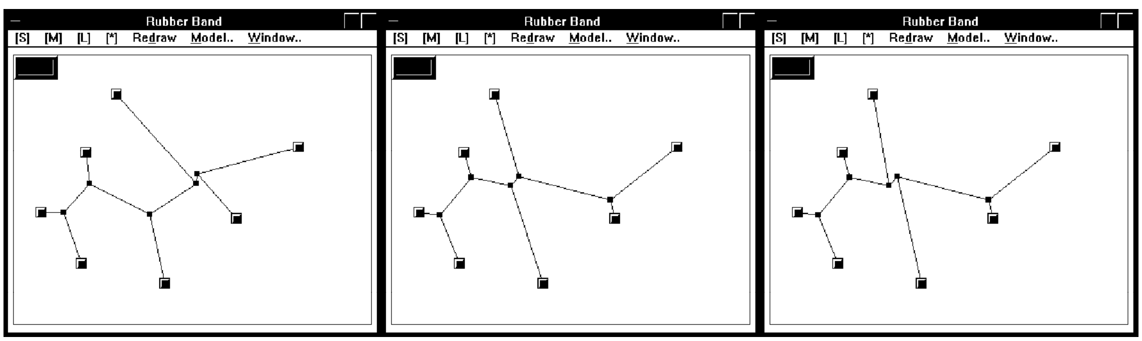


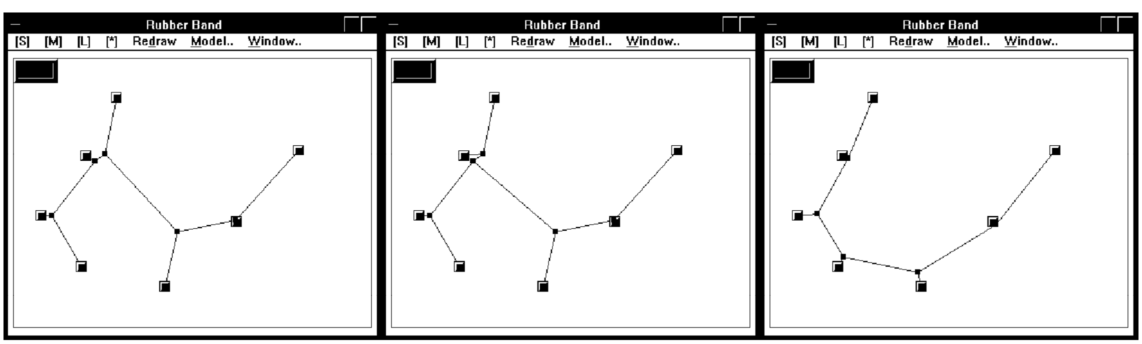
Fig 3 - Three states (left to right) in the iterative improvement of a Steiner tree. The solution (right) is the optimal for the given topology.

3. Improving the Topology

In some situations, when the initial topological is suboptimal, two or more Steiner points tend to hit one each other as illustrated in Fig-4 (center). In this case, we apply an swapping algorithm that tries to reorder the four wires connected to the two Steiner points. In many cases, this heuristics improved the topology.



(a) (b) (c)

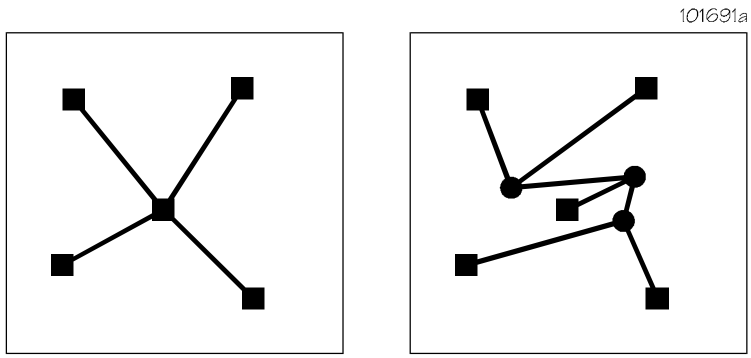


(d) (e) (f)

Fig 4 - Fixing the topology. The initial topology (a) is suboptimal, and the two Steiner points at the (b) hit one each other. After rearrangement of e four connection of the two movable points (c), a better topology and tree are achieved (d). By repeating this step, the final Steiner tree (f) is acheved.

4. Finding the topology

When a Steiner tree of a given set of points is required, the topology can be determined heuristically, and then the iterative algorithm is applied to find the optimal solution for that topology. We have experimented with the heuristic of finding the MST and then converting it to a Steiner tree by adding *m-1* Steiner points near every point of degree *m>1* (Figure 6), and have got promising results. Another possible approach, which is applicable for small trees (a common case in practical VLSI and MCM problems) is to enumerate the possible topologies and to choose the one that gives the best solution.



(a) (b)

Figure 5 - Converting an MST (a) into a full Steiner tree (b) by adding Steiner points in the neighborhood of points with degree greater than 1. In the full tree, the non movable points are of degree 1 and the movable are of degree 3.

5. Obstacles

Introduction of obstacles on the plane has a twofold effect on the problem: the position of the Steiner points is constrain to be outside the obstacles area, and the wire bending over the obstacles change the value of the surface function (Figure 6). The iterative algorithm can be extended to support obstacles as well by using homotopic transformation such that the topology is preserved and points can not be moved through obstacles area. The wires may bend around the obstacles as in a rubber-band sketch and the force applied by a wire on a point is in the direction of the first bent of the wire.

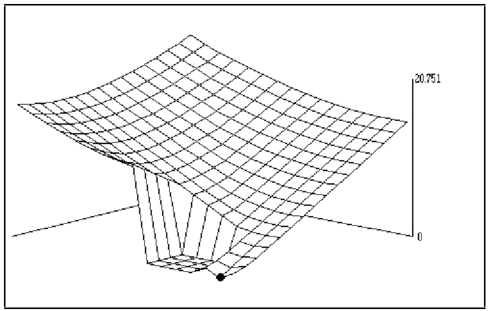


Fig 6 - The surface for a single non-moveable point (dark dot) near an obstacle (lower flat area). The length of the wire is calculated considering the bending around the obstacle.

6. Forest of Steiner Trees (Multiple Nets)

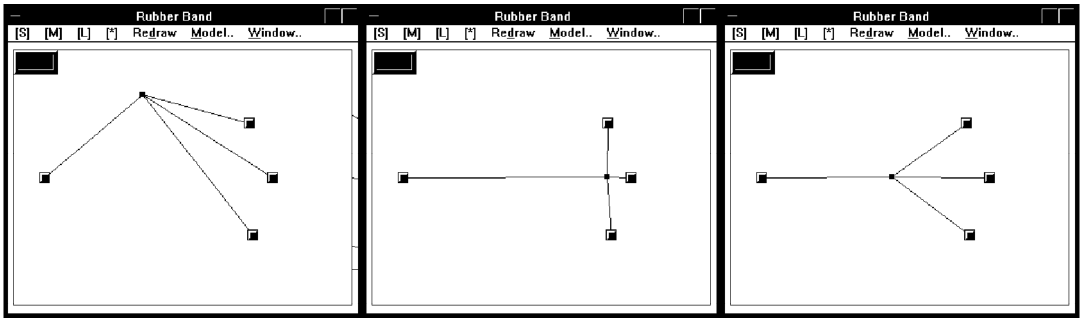
The iterative algorithm can be extended to simultaneously improve multiple nets (a Steiner forest). This is a common problem in VLSI and MCM interconnect. At each iteration, the new locations of the movable points are calculated in the same way as in the basic (Weber problem) case. For each net, the other nets are treated as obstacles. The forces on every moveable points include the forces of the incident as well as the attached (bent) wires.

7. Multi-Layer

The iterative algorithm supports multi-layer sketches and can consider the interaction between the layers. For example, a via between layer A and layer C can have attached nets on layers A, B, and C. The algorithm can consider the force applied on the via by the incident and attached nets on the three layers .

8. Constrains

Until now, we have discused a constant wire force function, which causes minimization of the total wiring length. Use of other force functions results in optimization of various properties of the interconnect, and can capture the notion of constrains imposed on the wiring. For example, use of a force function of the form a $F=KX$, where X is the length of the wire, will result in reduction of the length of the longest wire. By using different force functions for different nets, one can capture the criticality of certain nets. Use of a force function which is not monotonic or linear can represent upper and lower bounds on wire segment length. Figure 7 shows an example of two different results achieved using two different force functions.



(a) (b) (c)

Figure 7 - The effect of the force function on the final tree. When the initial tree (a) is improved using fixed forces, the total wire length is minimized (b). When a force function proportional to the l^3 (where l is the length of the wire segment) is used, the result (c) has longer total wiring but the longest segment is shorter.

9. Analysis of the Basic Case (Weber Problem)

9.1. Definitions

Let $\{P_i=(x_i, y_i) \mid i=1..n\}$ be the set of fixed points. Let $S=(x,y)$ be the movable (Steiner) point. Let V_i, $i=1..n$ be the directed segments from point S to points P_i. The length of V_i is $L_i=|V_i|$ and its direction is angle α_ι. The total length of the segments is

$$(0.1) \qquad L = \sum_{\iota} L_i$$

Let $\varepsilon > 0$ be an infinitesimally small, positive number such that $\varepsilon << V_i$. Let V_m be a movement vector of magnitude $|V_m|=\varepsilon$ and a direction α (Figure 8) such that the location of the point S after the movement by vector V_m is

$$(0.2) \qquad S^* = S + V_m$$

Let V_i^* be the (new) segments from S^* to the fixed points P_i, L_i^* be their lengths, and L^* the new total length:

$$(0.3) \qquad L^* = \sum_{i} L_i^*$$

The increase in the length of an individual segment by a movement in direction α is

$$(0.4) \qquad \Delta_i(\alpha)=L_i^*-L_i$$

And the increase in the total length is

$$(0.5) \qquad \Delta(\alpha) = L^{*}\text{-}L = \sum_{i} \Delta_{\iota}(\alpha)$$

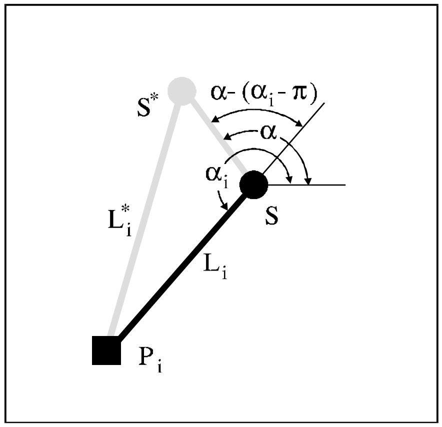


Figure 8 - A movable point S is moved in direction α to point S^*. L_i and L_i^* are the lengths of the wire segment that connects S to the fixed point P_i before and after the movement respectively. The analysis in this paper is done for movements of small distance $\varepsilon=|S-S^*|$ such that $0<\varepsilon<<1$.

9.2. Movement Analysis

The increase in the length of a single segment (Figure 8) by a small movement in direction α and magnitude ε is

$$(0.6) \qquad \Delta_i(\alpha)=\varepsilon \bullet cos(\alpha-(\alpha_i\text{-}\pi)).$$

Which is also

$$(0.7) \qquad \Delta_i(\alpha)=\varepsilon \bullet \{-cos(\alpha_i) \bullet cos(\alpha) - sin(\alpha_i) \bullet sin(\alpha)\}$$

By (0.3) and (0.7) the increase in the total length is

$$(0.8) \qquad \Delta(\alpha)=\varepsilon \bullet \{-A \bullet sin(\alpha) - B \bullet cos(\alpha)\}$$

Where

$$(0.9) \qquad A = \sum_{\iota} sin(\alpha_i) \,, \qquad B = \sum_{\iota} cos(\alpha_i)$$

Formula (0.8) can also be written as

$$(0.10) \quad \Delta(\alpha)=\varepsilon \bullet \{-K \bullet cos(\alpha-C)\}$$

Where

$$(0.8) \qquad C = atan(\frac{A}{B})$$

and

$$(0.9) \qquad K = \frac{B}{cos(C)} = \frac{A}{sin(C)}$$

By (0.10) the minimum of $\Delta(\alpha)$ is when $\alpha=C$ and its value at this point is $\varepsilon \bullet K$.

Conclusion: The maximal reduction in the total length by a movement of distance ε is when the movement is in direction $\alpha=C$ and the reduction is $\varepsilon \bullet K$.

9.3. Modeling as Forces

Let $\{F_i \mid i=1..n\}$ be a set of force of magnitude *1* and of direction α_i[1]that are applied by the 'rubber band' wire segments on the moveable point *S* (figure 2). Let *F* be the sum of the force vectors

$$(1.1) \qquad F = \sum_i F_i$$

The *X* and *Y* components of F_i are $cos(\alpha_i)$ and $sin(\alpha_i)$ respectively, and by (0.9), *B* and *A* are exactly the *X* and *Y* components of the combined vector *F*. By (0.8) and (0.9) C is the direction, and K is the magnitude of F.

We have shown that the maximum improvement is when the (infinitesimally small) movement is in direction *C* ,which is also the direction of *F*, and the improvement in this direction is proportional to K, which is the magnitude of *F*.

Conclusion: for infinitely small movements, the maximal improvement is when the moveable point is moved in the direction of the combined force of the forces applied on it. The improvement is proportional to the magnitude of this force.

9.4. Multiple steps are required

The optimal solution is not necessarily in the direction of the combined force F. If the movements are done in the direction of the highest local improvement, multiple steps may be require (Figure 9).

[1] The direction of the force is the same direction as the vector from the moveable to the fixed point.

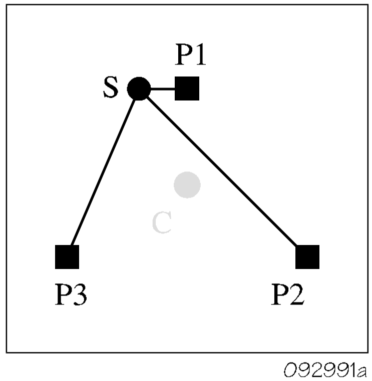


Figure 9 - An example where the equivalent force is not in the direction of the optimal solution. The distance between P1 and S is infinitsimally small, still the direction of the equivalent force is not infinitesimally close to 270^{o} (south) where the optimal solution is but it is more to the right (south east).

9.5. Analysis of the Surface's Minimum Points

Solving the Weber problem is equivalent to finding a minimum point of the surface *L*. This surface has properties that enable the use of iterative greedy improvement to find its minimum.

Lemma : For every instance of Weber problem, the total length function *L(x,y)* satisfies:

1. L has a global minimum.
2. Every local minimum of *L* is also a global minimum.
3. Between two local minimums, there is a path that includes only local minimum points.

Property 1 is true because *L*(x,y) is continuous and has upper and lower bounds on every (finite) closed domain. When $|x|+|y|$ goes to infinity, *L(x,y)* goes to plus infinity, and must have a minimum (i.e. the minimum is not on the infinite boundary of the plane).

The proof of 2 and 3 is based on a reduction to a one dimensional problem. It is sufficient to show that properties 2, 3 are true for L over *every* cut of the plan by a straight line.

Definition: A *bimonomoton function* is a function $Y=f(x)$, that is

1. Continuous.
2. Piecewise derivable, with finite number of pieces.
3. The derived function Y' is non decreasing such that $x_1>x_2 \Rightarrow Y'(x_1) \geq Y'(x_2)$.
4. There are two numbers $x_1 \leq x_2$, such that $Y(x)$ is constant in $[x_1,x_2]$, and $Y'(x)$ is negative for $x<x_1$ and is positive for $x>x_2$. The range $[x_1,x_2]$ is called *the center* of Y.

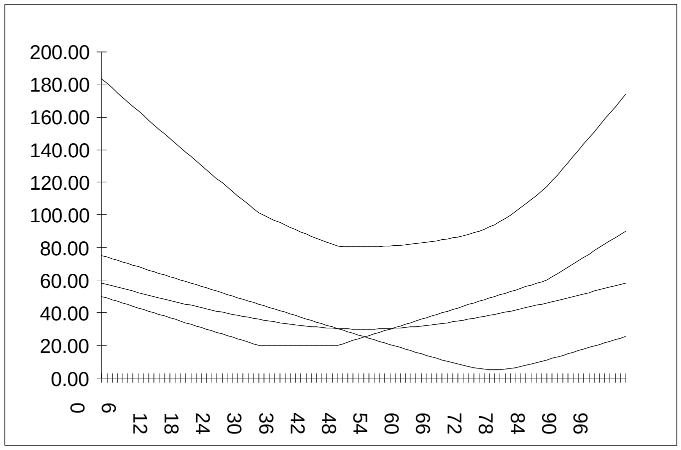


Fig 10 - Bimonotone functions. The upper one is the sum of the other three. In one of the functions, $x_1 < x_2$ while in the other three, $x_1 = x_2$.

On a bimonotone function, every local minimum is also a global minimum - That's clear since the function is continuous and the piecewise derived function is non decreasing.

On a bimonotone function a point between two minimum point is also a minimum point - That's clear since the function is continuos and the piecewise derived function is non decreasing.

Sum of a finite number of bimonotone functions is also a bimonotone function - Let f_a, f_b be two bimonoton functions with centers $[x_{1a}, x_{2a}]$, $[x_{1b}, x_{2b}]$. Let $f = f_a + f_b$ be the sum function. Since f_a, f_b are bimonotone, they are continuos, piecewise drivable, and have derivative functions which are non decreasing, and that hold for f as well. The center of f is the range which f' is positive above it and negative below it. Since f' is non decreasing, negative below the union of the two centers, and positive above it, such a range exist (and there is exactly one).

***L_i* over a cut is bimonomoton** - Let H be a line on the plane, and let $P_i = (x,y)$ be a non movable point. We can assume without loss of generality that the cut is the line $y=0$, and that P_i is on the line $x=0$ (otherwise a rotation and translation can be used). $L_i(x)$ over the cut H is

$$(2.1) \qquad L_i(x) = \sqrt[2]{y^2 + x^2}$$

Where y is the y value of point P_i. By this, function L_i is bimonotone for every value of y.

***L* over a cut is bimonotone** - We have shown that L_i is bimonotone over every cut and that sum of bimonotoe functions also bimonotone. Since L is the sum of L_i, L is also bimonotone.

Conclusion - L is bimonotone on every cut and as such, every local minimum is also a global minimum and the points between two minimum points are also minimum points. This is sufficient that the same is true for L over the entire plane.

10. Future Work

Further research on the behavior of a multiple nets, multi layer system with obstacles is required. Such a research can suggests better improvement methods that will converge faster, and ways to bound the extra cost of the approximation compared to the optimal solution. In addition, the effect of various force function needed to be studied. Non fix force functions can be used to optimization of various properties of the interconnect such as bounded wire length, but some cost functions are inappropriate and do not guaranty convergence.

11. Conclusion

By modeling the rubber band sketch as a physical system of springs and using a simple iterative improvement algorithm, the wiring cost can be improved. This technique is applicable for multi-layer, multi-nets, rubber-band sketches with obstacles. Variations on the force function can be used to direct the algorithm to a more desirable solution according constrains imposed on the wiring.

12. References

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