

Partial Reluctance K Based Circuit Analysis Is Stable

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ABSTRACT

Although the stability of circuit analysis based on partial reluctance K has been proved for parallel bus structures [5] and the structures with sufficiently discretized conductors [4], the stability for general interconnect topologies has not been proved. This paper proved that full partial reluctance matrix and sparsified partial reluctance matrix obtained by ignoring small mutual terms in partial reluctance extraction is positive definite for any interconnect structures, which is the necessary and sufficient condition for the circuit analysis based on partial reluctance to be stable.

1. INTRODUCTION

On-chip inductance effect is becoming increasingly important due to the increase of clock speed and the decrease of wire resistance by using copper technology and hierarchical wire width. Capturing on-chip inductance effect is very difficult due to unknown circuit return path. The concept of partial inductance proposed by Rosa almost a century ago [11] was introduced by Ruehli to the circuit design field in 1972 [12]. Partial inductance approach avoids the unknown loop problem by assuming virtual loop closed at infinity. However, due to the virtual infinity loop, the mutual partial inductance couplings are now among all the conductor segments and the resulting partial inductance matrix is extremely dense. Further more, if we sparsify the matrix by simply truncating the small mutual entries, the resulting partial inductance matrix may not be positive definite and thus may be unstable in circuit analysis.

Several methods have been proposed to sparsify partial inductance matrix while preserving the circuit stability by making some assumptions on the current return path, for example, the “shift-truncate” method [8] and “return-limited loop inductance” concept [13]. However, the accuracy of these approaches may vary because the current return path assumptions depend on the interconnect structure.

Partial reluctance circuit model, or “K-based method”, was first proposed by Devgan *et al* in 2000 [5]. It captures the on-chip inductance effect by extracting partial reluctance, which has locality similar to capacitance.

Partial reluctance was first called K element in [5], while the term “susceptance” is used for K element in [3] and the term “reluctance” is used in [4]. However, the term “partial reluctance” is more appropriate to represent the inversion of partial inductance matrix since the definition of “susceptance” is the imaginary part of admittance and “reluctance” is the reciprocal of inductance [14]. *Inductance* is different from *partial inductance*, since the former is a property of closed loops while the latter is a property of segments. Similarly, *partial reluctance* is not *reluctance*. If we say *inductance* is a flux controlled non-linear function of flux vs. current, *reluctance* is a current controlled non-linear function of flux vs. current. *Partial reluctance* is the inverse of *partial inductance* and K element is *partial reluctance*. Similar relationship can be found between *capacitance* and *elastance*: *capacitance* is a charge controlled non-linear function of charge vs. voltage and *elastance* is a voltage controlled non-linear function of charge vs. voltage.

Also, later proposed circuit models “VPEC” model in [10] [15] and “wire duplicate” model in [16] are also shown to be equivalent to partial reluctance model.

Experiments in [5, 3, 2, 7, 4, 15, 16] showed that partial reluctance has locality so that the faraway mutual partial reluctance are much smaller compared with mutual partial inductance. By ignoring small mutual reluctance terms, the inductance effect can be captured more effectively.

Because most of the off-diagonal entries of the partial reluctance matrix are negative, it is widely believed that the full and truncated partial reluctance matrices are positive definite so that the partial reluctance based circuit analysis is stable. Devgan *et al* [5] showed it to be true for parallel conductors with equal lengths because all the mutual partial reluctance terms in the equal length structure are negative. However, there may be positive off-diagonal entries in the partial reluctance matrix of conductors with unequal lengths [4] and the positive definiteness stayed unknown for general structures.

Chen *et al* proved that “the reluctance matrix, K , is diagonally dominant and symmetric positive definite when all the conductors are sufficiently discretized” in [4]. But *sufficiently discretize* all the conductors introduces too many conductor segments thus hurts the performance significantly. RBC (Recursive Bisection Cutting) algorithm proposed in [4] cuts the interconnect conductors and extract partial reluctance adaptively to diminish positive mutual partial reluctance. Although RBC algorithm is guaranteed to obtain the partial reluctance matrix with only negative off-diagonal terms, the extraction

process will probably run many times because cutting long conductors to diminish one mutual partial reluctance may introduce new positive mutual reluctance. Appendix A shows a simple example that the RBC algorithm will run the extraction process too many times to diminish positive mutual terms in partial reluctance matrix and hence result in low efficiency.

As it is well known that all off-diagonal entries of a matrix being negative while the diagonal entries are positive is a sufficient condition for the matrix being positive definite but not a necessary condition. In this paper, we prove that even with positive off-diagonal entries, the partial reluctance matrix is still positive definite and the partial reluctance based circuit simulation is stable. With the proof of stability of partial reluctance for general interconnect structure, the efficiency of partial reluctance approach will not be sacrificed by the discretization algorithms.

Before presenting the proof of the stability of partial reluctance based simulation, we will state the algorithm for extracting and sparsifying partial reluctance matrix K in Section 2. Then we prove that the full partial reluctance matrix is positive definite in Section 3 and prove that the truncated partial reluctance matrix is positive definite in Section 4. We will give concluding remarks in Section 5.

2. OBTAINING SPARSE PARTIAL RELUCTANCE MATRIX

In [5], partial reluctance matrix K is defined as the inverse of partial inductance matrix L .

$$[K] = [L]^{-1} \quad (1)$$

And from the view of K 's physical meaning, another definition of K matrix can be stated as in [7]: "The element K_{ij} is the current flowing through the i th conductor when the magnetic vector potential drop along all conductors, except the j th, are set to zero, and the magnetic vector potential drop along the j th conductor is raised to unit potential." This definition is showed to be equivalent to the definition in Eq. 1 and Eq. 2 can illustrate it clearer.

$$\begin{bmatrix} K_{11} & K_{12} & \cdots \\ K_{21} & K_{22} & \cdots \\ \vdots & \vdots & K_{nn} \end{bmatrix} \begin{bmatrix} \int A_1 dl_1 \\ \vdots \\ \int A_n dl_n \end{bmatrix} = \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} \quad (2)$$

From the two definitions of partial reluctance, there are two approaches for extracting partial reluctance matrix K . One is to inverse the partial inductance matrix obtained by inductance extraction tool while the other is to put unit vector potential drop integration along the aggressive conductor i and let vector potential drop be zero along other conductors when the i th column of K matrix is being extracted.

If skin-effect and proximity-effect are ignored and each conductor segment is viewed as only one conductor filament, the above two approach for partial reluctance extraction are equivalent. We will show that the full partial reluctance matrix obtained by reluctance extraction is positive definite in Section 3.

Although circuit simulation using full partial reluctance matrix converges much faster than full partial inductance matrix due to its locality, the major speed up by K based circuit analysis is because small mutual partial reluctance can be truncated. Experimental results in [5, 3, 2, 7, 4, 16, 15] verified that truncating small mutual reluctance terms will enhance the speed of inductive circuit simulation by at least two orders of magnitude with little accuracy loss. It is also needed to prove the positive definiteness of partial reluctance matrix after truncating small mutual terms for general cases to expand the conclusion of above papers to more complicated interconnect structures from parallel bus structure.

Before we start to prove the stability of circuit simulation based on partial reluctance, it is necessary to describe the truncated partial reluctance matrix extraction process:

1. Set conductor segment i as aggressor and put it into set M_i .
2. Ignore conductor segments j that the mutual reluctance between conductor i and j are small enough to be truncated.
3. Put the conductors segments that have not been truncated in set M_i .
4. Let $\int A_i dl_i = 1$.
5. Let $\int A_j dl_j = 0$, $j \in M_i$ and $j \neq i$.
6. Calculated the current distribution on conductor i and j ($j \in M_i$).
7. The i^{th} column of the sparse partial reluctance matrix is obtained: K_{ij} is the the current on conductor j ($j = i$ or $j \in M_i$), $K_{ij} = 0$ if $j \notin M_i$.
8. Choose another conductor as aggressor and go to step 1 to fill another column of the partial reluctance matrix K .
9. Let $K = \frac{1}{2} \cdot (K + K^T)$ to make the sparse partial reluctance matrix symmetric.

3. FULL PARTIAL RELUCTANCE MATRIX IS POSITIVE DEFINITE

It is shown that full partial inductance matrix is positive semi-definite although truncated partial inductance matrix will not keep positive semi-definite [8]. In the following, we will prove full partial reluctance matrix is positive definite.

Consider a system with n conductor segments, the energy stored in a static magnetic field equals:

$$\mathcal{E}_S = \frac{1}{2} \int j \cdot A dV \quad (3)$$

where the integration includes all the regions with non-zero current j . The energy $\mathcal{E}_S$ can be also represented by the magnetic flux density B [9],

$$\mathcal{E}_S = \frac{1}{2\mu_0} \int B^2 dV \quad (4)$$

Then we have $\mathcal{E}_S > 0$ because B is not always zero in the space.

Suppose the integration of vector potential drop along the conductor segments are $\int A_1 dl_1, \int A_2 dl_2, \dots, \int A_n dl_n$ and the current on the conductor segments are $I_1, I_2, \dots, I_n$.¹ Assume the current density is zero outside the conductors, $\mathcal{E}_S$ can be expressed as

$$\int j \cdot A dV = \sum_{i=1}^n I_i \int A_i dl_i \quad (5)$$

Let $[A] = (\int A_1 dl_1, \int A_2 dl_2, \dots, \int A_n dl_n)^T$ and $[I] = (I_1, I_2, \dots, I_n)^T$, Eq. 5 can be written in vector form

$$2 \cdot \mathcal{E}_S = \int j \cdot A dV = [I^T] \cdot [A] \quad (6)$$

With the definition and physical meaning of partial reluctance in Eq. 2, Eq. 6 will be

$$2 \cdot \mathcal{E}_S = [A^T] \cdot K^T \cdot [A] = [A^T] \cdot K \cdot [A] \quad (7)$$

We can apply the current on the conductor segments to make the flux $[A]$ be any n -dimension vector, so full partial reluctance matrix K is positive definite since

$$[A^T] \cdot K \cdot [A] > 0 \quad (8)$$

The proof in this section can be concluded as theorem 1:

THEOREM 1. *The full partial reluctance matrix is positive definite.*

4. TRUNCATED PARTIAL RELUCTANCE MATRIX IS POSITIVE DEFINITE

Based on theorem 1 proved in the previous section, we will prove that sparse (or truncated) partial reluctance matrix is also positive definite by induction. Here, we introduce a new term “Minkowski Matrix” [6], and it is defined as:

DEFINITION 1. *If all eigenvalues of a square matrix is positive, it is a Minkowski Matrix.*

Perceptibly, **positive definite matrix is a symmetric Minkowski Matrix**. Since the truncated partial reluctance may not keep symmetric during our proof, it is appropriate to use Minkowski Matrix instead of positive definite matrix.

We first prove that when only one mutual partial reluctance between two conductors is ignored in reluctance extraction process, the truncated partial reluctance matrix is still a Minkowski Matrix. We are using the following lemma to prove the truncated partial reluctance matrix to be a Minkowski Matrix:

LEMMA 1. *A square matrix is a Minkowski Matrix if and only if all the determinants of K 's principal minors are positive. [6]*

Here “principal minor” means a sub square matrix on the main diagonal.²

We use the following notation in the proof:

- K : the partial reluctance matrix of n conductor segments without truncation.
- L : inversion of K , the full partial inductance matrix of n conductor segments.
- $\mathcal{K}$: the truncated partial reluctance matrix of n conductor segments.

¹By magneto-quasi-static approximation, we can ignore the inner product of current and flux which are not on the direction along the conductor segments

²An $n \times n$ matrix has n principal minors. For example, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, a_{11} and A itself are principal minors of A but a_{22} is not

- $\mathcal{L}$: inversion of $\mathcal{K}$.
- K' : the partial reluctance matrix of $n - 1$ conductor segments without sparsification.
- L' : inversion of K' .
- K_{ij} : the entry (i, j) of matrix K .
- $\det(K)$: the determinant of matrix K .
- $\bar{K}_{ij}$: the matrix by deleting the i th row and j th column from K .
- $< K >_{ij}$: the cofactor of K , which is $(-1)^{i+j} \det(\bar{K}_{ij})$.

As theorem 1 in Section 3, K is positive definite, then it is a Minkowski Matrix. So we know that the determinants of K 's sub square matrices on the main diagonal are all positive, which is:

$$\left\{ \begin{array}{l} K_{11} > 0 \\ \det \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} > 0 \\ \vdots \\ \det \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n-1} \\ K_{21} & K_{22} & \dots & K_{2n-1} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n-11} & K_{n-12} & \dots & K_{n-1n-1} \end{bmatrix} > 0 \\ \det(K) > 0 \end{array} \right. \quad (9)$$

Suppose the mutual partial reluctance K_{1n} is ignored and we obtain the truncated $n \times n$ partial reluctance matrix $\mathcal{K}$. If the ignored mutual reluctance is not K_{1n} , we can always renumber the conductor segments, swap the rows and columns of K and $\mathcal{K}$ to let the ignored mutual reluctance numbered as K_{1n} without changing the nature of the system. When extraction the n th column of $\mathcal{K}$, we remove conductor segment 1 first, then assume unit vector potential drop along conductor segment n and calculate the current distribution on conductor 2, ..., n . It should be noticed that not only $\mathcal{K}_{1n} = 0$ is different, $\mathcal{K}_{2n}, \dots, \mathcal{K}_{nn}$ are all different from $K_{2n}, \dots, K_{nn}$ because of the shielding effect of partial reluctance.

Since no conductor segment is removed when columns of $\mathcal{K}$ are being calculated except column n , the determinants of $\mathcal{K}$'s main diagonal sub-matrices keeps the same as K 's except $\det(\mathcal{K})$. That is

$$\left\{ \begin{array}{l} \mathcal{K}_{11} = K_{11} > 0 \\ \det \begin{bmatrix} \mathcal{K}_{11} & \mathcal{K}_{12} \\ \mathcal{K}_{21} & \mathcal{K}_{22} \end{bmatrix} = \det \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} > 0 \\ \vdots \\ \det \begin{bmatrix} \mathcal{K}_{11} & \mathcal{K}_{12} & \dots & \mathcal{K}_{1n-1} \\ \mathcal{K}_{21} & \mathcal{K}_{22} & \dots & \mathcal{K}_{2n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{K}_{n-11} & \mathcal{K}_{n-12} & \dots & \mathcal{K}_{n-1n-1} \end{bmatrix} \\ = \det \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n-1} \\ K_{21} & K_{22} & \dots & K_{2n-1} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n-11} & K_{n-12} & \dots & K_{n-1n-1} \end{bmatrix} > 0 \end{array} \right. \quad (10)$$

With Eq. 10, we only need to show $\det(\mathcal{K}) > 0$ to prove that $\mathcal{K}$ is a Minkowski Matrix. The determinant of $\mathcal{K}$ can be calculated by “determinant expansion” [1]:

$$\begin{aligned} \det(\mathcal{K}) &= \sum_{i=1}^n \mathcal{K}_{i\ n} \cdot \langle \mathcal{K} \rangle_{i\ n} \\ &= \sum_{i=1}^n \mathcal{K}_{i\ n} \cdot \langle K \rangle_{i\ n} \end{aligned} \quad (11)$$

where $\langle \mathcal{K} \rangle_{i\ n}$ and $\langle K \rangle_{i\ n}$ are cofactors of $\mathcal{K}_{i\ n}$ and $K_{i\ n}$, and they are identical since the first $n - 1$ columns of $\mathcal{K}$ and K are the same. Recall the relationship of cofactor and inversion [6], we have

$$L_{n\ i} = \frac{\langle K \rangle_{i\ n}}{\det(K)} \quad (12)$$

Also because $\mathcal{K}_{1\ n}$ is truncated, we know $\mathcal{K}_{1\ n} = 0$. So the determinant of $\mathcal{K}$ equals

$$\begin{aligned} \det(\mathcal{K}) &= \det(K) \cdot \sum_{i=1}^n \mathcal{K}_{i\ n} \cdot L_{n\ i} \\ &= \det(K) \cdot \sum_{i=2}^n \mathcal{K}_{i\ n} \cdot L_{n\ i} \end{aligned} \quad (13)$$

By removing conductor 1, we obtain an $(n - 1)$ -conductor system and its partial reluctance matrix K' . In the $(n - 1)$ -conductor system, conductor i is identical with conductor $i + 1$ of the previous n -conductor system. So the n th column of $\mathcal{K}$ can be mapped to the $(n - 1)$ th column of K' as

$$\mathcal{K}_{i\ n} = K'_{i-1\ n-1}, \text{ where } i = 2, \dots, n \quad (14)$$

And the inversion of K' , the partial inductance matrix L' , will have the following relation with K'

$$\sum_{i=1}^{n-1} K'_{i\ n-1} \cdot L'_{n-1\ i} = 1 \quad (15)$$

From the physical nature of partial inductance, we know that partial inductance dose not have shielding effect when proximity effect is not significant enough. Then the mutual partial inductance will keep unchanged when other conductors is removed from the system. Thus, for L and L' , we have

$$L_{n\ i} = L'_{n-1\ i-1}, \text{ where } i = 2, \dots, n \quad (16)$$

Combine Eq. 13, Eq. 14, Eq. 15 and Eq. 16, the determinant of $\mathcal{K}$ will be

$$\begin{aligned} \det(\mathcal{K}) &= \det(K) \cdot \sum_{i=2}^n \mathcal{K}_{i\ n} \cdot L_{n\ i} \\ &= \det(K) \cdot \sum_{i=2}^n K'_{i-1\ n-1} \cdot L'_{n-1\ i-1} \\ &= \det(K) \cdot \sum_{i=1}^{n-1} K'_{i\ n-1} \cdot L'_{n-1\ i} \\ &= \det(K) \end{aligned} \quad (17)$$

And because $\det(K) > 0$, we also proved that $\det(\mathcal{K}) > 0$. Together with Eq. 10, we know that truncated partial reluctance matrix by ignoring mutual partial reluctance $\mathcal{K}_{1\ n}$ is a Minkowski Matrix through lemma 1. To make the partial reluctance matrix symmetric, we need to truncate $\mathcal{K}_{n\ 1}$. We swap column 1 and n of $\mathcal{K}$ and then swap row 1 and n of $\mathcal{K}$, and it is still a Minkowski Matrix because of the following lemma:

LEMMA 2. *Swapping column i and j of a square matrix K and then swapping row i and j of K , the determinant and eigenvalues of K will keep unchanged.*

By induction, if the partial reluctance matrix with more than one entry are truncated, the matrix is a Minkowski Matrix because we can truncate one more mutual partial reluctance entry each step, swap the columns and rows by lemma 2 and keep the matrix to be Minkowski Matrix. So we have:

LEMMA 3. *The sparse partial reluctance matrix, obtained through the extraction process step 1 to 8 in Section 2, is a Minkowski Matrix.*

Recall that we will symmetrize the sparse partial reluctance matrix $K = \frac{1}{2} \cdot (K + K^T)$ at the last step of extraction. It is easy to find that the K^T is also a Minkowski Matrix and its eigenvalues are all the same as K . So the symmetric sparse partial reluctance matrix $\frac{1}{2} \cdot (K + K^T)$ is also a Minkowski Matrix, hence it is a positive definite matrix.

THEOREM 2. *The sparse partial reluctance matrix, obtained through the extraction process in Section 2, is positive definite.*

By theorem 2, we know the circuit model constructed from the sparse partial reluctance matrix is passive, and hence the partial reluctance based circuit model is stable in simulation for general interconnect structure.

In paper [16], the circuit model of larger window with multiple aggressors is proposed to enhance the efficiency by reuse the computation resource. Although the partial reluctance matrix was not explicitly given out, there is still an equivalent partial reluctance matrix for the circuit model in the paper. From theorem 2, corollary 1 can be easily deduced and then the “group” method, using larger window with multiple aggressors, is also stable for general interconnect structures. However, select optimal group window size for unequal length conductor structure will not be as easy as that for bus structure discussed in [16].

COROLLARY 1. *The sparse partial reluctance matrix, obtained through “group” method proposed in [16] is positive definite.*

5. CONCLUSION

In this paper, we proved that both full partial reluctance matrix and sparse partial reluctance matrix having small mutual reluctance truncated are positive definite. Then the stability of the partial reluctance approach is guaranteed not only in parallel bus structure but also in general interconnect structures.

With the proof in this paper, instead of running the K extraction algorithm several times to find a sparsified K matrix without positive off diagonal terms, run extraction algorithm only once is enough to have a stable circuit model for circuit analysis.

Together with K 's locality, the conclusions in paper [5, 7, 3, 2, 4, 16, 15] that the partial reluctance approach is efficient, practical and stable, can be generalized from parallel bus structure or sufficiently discretized structure to any interconnect structure.

6. REFERENCES

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APPENDIX

A. AN INTERCONNECT EXAMPLE TO ILLUSTRATE RBC ALGORITHM

Here is an example that the RBC algorithm (Recursive Bisection Cutting Algorithm) may spend too many iterations of partial reluctance extraction to guarantee against positive mutual partial reluctance. Suppose we have two short signal lines near a bus structure and the interconnect wires are cut short enough to obtain accurate result. Figure 1 shows only a part of the structure we are interested and the calculated partial reluctance matrix corresponding to that part is shown in Eq. 18. The dimension of all the wires in this example are shown in the figures. The unit for the partial reluctance entries in Eq. 18 is $10^7 H^{-1}$.

$$\begin{bmatrix} 345 & \mathbf{13} & -108 & 0 & 0 & 0 \\ \mathbf{13} & 345 & -108 & 0 & 0 & 0 \\ -105 & -105 & 214 & -100 & 0 & 0 \\ 0 & 0 & -100 & 217 & -100 & 0 \\ 0 & 0 & 0 & -100 & 217 & -100 \\ 0 & 0 & 0 & 0 & -102 & 152 \end{bmatrix} \quad (18)$$

It can be noticed that the mutual partial reluctance $K_{12} = K_{21} = 1.3 \times 10^8 > 0$ due to the unequal length of the conductor segments. To guarantee the mutual reluctance entries to be negative, RBC algorithm is performed and the bus line conductor segment closes to the signal wire segments is cut by half which is shown in Figure 2.

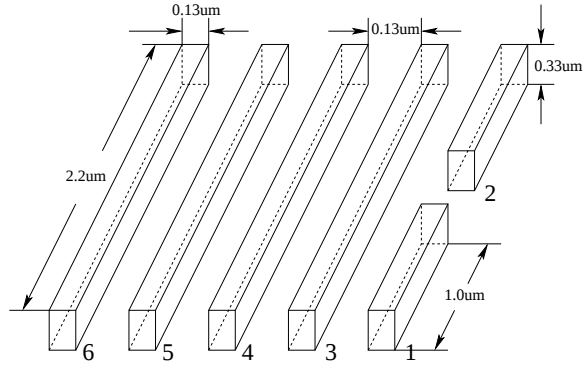


Figure 1: A part of a bus structure and nearby interconnect

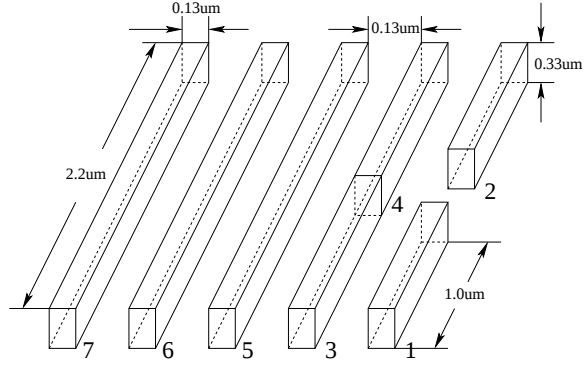


Figure 2: Figure 1 after one iteration of RBC algorithm

$$\begin{bmatrix} 392 & -40 & -207 & 1.5 & 0 & 0 & 0 \\ -40 & 392 & 1.5 & -207 & 0 & 0 & 0 \\ -204 & \underline{3.9} & 417 & -11 & -95 & 0 & 0 \\ \underline{3.9} & -204 & 417 & -11 & -95 & 0 & 0 \\ 0 & 0 & -96 & -96 & 215 & -100 & 0 \\ 0 & 0 & 0 & 0 & -100 & 217 & -100 \\ 0 & 0 & 0 & 0 & 0 & -102 & 152 \end{bmatrix} \quad (19)$$

Unfortunately, the cutting introduces new positive mutual reluctance K_{32} and K_{41} in Eq. 19 so that the longest conductor in current window will be cut by half and the extraction process will have to run again.

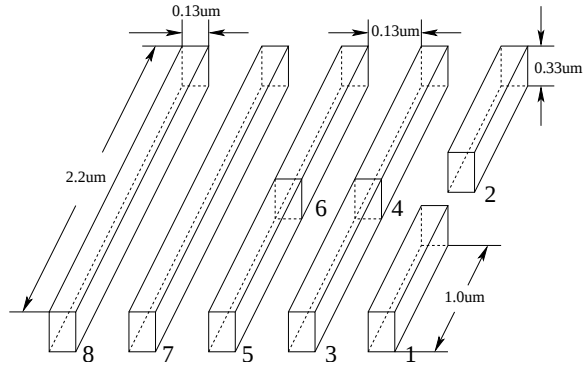


Figure 3: Figure 1 after two iterations of RBC algorithm

$$\begin{bmatrix}
392 & -40 & -207 & 1.5 & 0 & 0 & 0 & 0 \\
-40 & 392 & 1.5 & -207 & 0 & 0 & 0 & 0 \\
-207 & \underline{\mathbf{6.8}} & 469 & -67 & -199 & \underline{\mathbf{17}} & 0 & 0 \\
\underline{\mathbf{6.8}} & -207 & -67 & 469 & \underline{\mathbf{17}} & -199 & 0 & 0 \\
0 & 0 & -196 & \underline{\mathbf{14.1}} & 419 & -12 & -95 & 0 \\
0 & 0 & \underline{\mathbf{14.1}} & -196 & -12 & 419 & -95 & 0 \\
0 & 0 & 0 & 0 & -96 & -96 & 215 & -100 \\
0 & 0 & 0 & 0 & 0 & 0 & -102 & 152
\end{bmatrix} \quad (20)$$

The cutting by RBC algorithm brings new positive mutual K again and thus another extraction process will be performed again. In this example, the iteration will not stop until all the bus wires are cut. If the bus is composed by 128 parallel conductors, the extraction algorithm will run 128 times, which is not acceptable.