Locality and Direct Extraction of Partial Reluctance K

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Abstract— This paper provides a method to directly extract K element, or *partial reluctance* from the physical meaning of partial reluctance. The approach of direct K extraction reveals the locality nature of partial reluctance and explains why inductance effect can be captured by sparse K matrix without knowing the current loop first. We use a 3-D discretization of a magneto-quasi-static integral formulation to simulate the current distribution and get the frequency dependent partial reluctance matrix K and resistance R. Directly extracting partial reluctance gives a clearer view of partial reluctance's physical meaning and avoids inverting the partial inductance matrix before running simulation and reduction based on RKC model. Also, benefiting from the locality of the partial reluctance, a hierarchical shielding technique is used in the extraction and it brings a great speed up. The result from examples shows advantages in direct partial reluctance extraction with shielding techniques over the traditional partial reluctance extraction by inverting L.

I. Introduction

As clock speed increases and less resistive wires are used to enhance on-chip signal propagation speed, inductance effect of on-chip interconnect is becoming more and more important. However, capturing on-chip inductance effect is very difficult because of unknown circuit return path. Ruehli introduced the concept of partial inductance developed by Rosa [12] to the circuit design field [13] to avoid the unknown loop problem. But since the partial inductance is based on the virtual loop closed at infinity, the coupling are now among all the conductor segments so that the resulting partial inductance matrix is extremely dense. Due to this global effect of dense partial inductance matrix, people are facing many difficulties on extracting on-chip parasitic parameter and simulating on-chip interconnect with RLC model.

Many work had been done to make the partial inductance sparse, such as the "shift-truncate" method [11] and "return-limited loop inductance" concept [14]. However, the accuracy of these approaches are not guaranteed under different interconnect topology because current return paths need to be assumed prior to the extraction while the return paths may not be true.

Fortunately, the inverse of partial inductance matrix, the *partial reluctance* matrix, has locality and stability [5]. The new circuit element, the partial reluctance, or K element, was first introduced by Devgan *et al* [5]. In [8], Ji *et al* developed a new circuit simulation tool, *KSim*, by incorporating the *K* element. Since the locality and stability of partial reluctance matrix *K*, KSim can achieve much higher performance than today's circuit simulation tools based on RLC model.

Other approaches based on the concept of partial reluctance such as "double inversion" by Beattie *et al* in [2] and "inductwise" by Chen *et al* in [4] are also proposed and great advantages over RLC model are shown in their work. And Zheng *et al* had done some work on model order reduction for RKC model in [15].

Ji *et al* illustrated the locality and positive definiteness of partial reluctance for parallel equal length bus structures in [8]. But as pointed out by Chen *et al* in [4], some off-diagonal reluctance terms may be positive in unequal lengths parallel conductor hence the proof of K's locality and stability by Ji *et al* will not be valid for general cases. We will show the locality of partial reluctance from the view of direct K extraction, or the physical meaning of partial reluctance.

The previous approaches [4] [8] to extract partial reluctance are to divide the circuit into some smaller parts, get the partial inductance matrices *L* of each one from available extraction tool such as FastHenry [10], inverse the *L* matrices to the *K* matrices and combine the small *K* matrices to a big one for the input of RKC simulation tools. It is clear that the matrix inversion is costly, and developing extraction method for directly catching partial reluctance becomes necessary.

Also, the partial reluctance shares many features with capacitance because one uni-directional magnetic problem can be transformed in to an electric problem [4]. So direct partial reluctance extraction may utilize some techniques used in capacitance extraction and gain the advantage over inductance extraction. This paper uses the hierarchical shielding algorithm, which was similar to the technique used in capacitance extraction [9], to accelerate the partial reluctance extraction.

The following section will give an overview about the background of partial reluctance. Section III illustrates the locality of partial reluctance by from its direct extraction approach, or K's physical definition. Section IV discusses some details of partial reluctance direct extraction, such as skin effect in high frequency. Hierarchical shielding algorithm to accelerate partial reluctance extraction is presented in section V and the experimental results are given in section VI to show the advantage of direct K extraction. Finally, section VII concludes this paper.

II. Physical Background Of Partial Reluctance

In [5], partial reluctance matrix K is defined as the inverse of partial inductance matrix L.

$$[K] = [L]^{-1} \tag{1}$$

The magnetic vector potential, A, is defined as

$$\begin{array}{c} \nabla \times A = \mu H \\ \nabla \cdot A = 0 \end{array} \tag{2}$$

Then by magneto-quasistatic approximation [6] and sinusoidal steady-state assumption, the vector potential has the following relation to the current distribution $J(r)$

$$A(r) = \frac{\mu_0}{4\pi} \int_{V'} \frac{J(r')}{\|\vec{r}_i - \vec{r}_j\|} dv' \tag{3}$$

where V' is the volume of all conductors. Then A_{ij}, the vector potential on conductor segment i due to the current on conductor segment j is

$$A_{ij} = \frac{\mu_0}{4\pi a_j} \int_{a_j} \int_{l_j} \frac{I_j d\vec{l_j}}{\|\vec{r}_i - \vec{r}_j\|} da_j \tag{4}$$

where l_j and a_j are the length and the cross section area of filament j. I_j is the current on conductor segment j. And A_i, the vector potential on conductor segment i due to all the current in the system, is

$$A_i = \sum_{j=1}^{n} A_{ij} \tag{5}$$

Also we know that with magneto-quasistatic approximation, the partial inductance between two conductor segments i and j is as follows [13]

$$L_{ij} = \frac{\mu_0}{4\pi a_i a_j} [\int_{a_i} \int_{a_j} \int_{l_i} \int_{l_j} \frac{d\vec{l_i} d\vec{l_j}}{\|\vec{r}_i - \vec{r}_j\|} da_i da_j] \tag{6}$$

With the definition of magnetic vector potential in Eq. (2) and zero transverse current on the conductors, A_i distributes uniformly on the cross-section of conductor segment i. So the magnetic vector potential along a conductor segment can be simply wrote down as

$$\frac{1}{a_i} \int \int A_i da_i dl_i = \int A_i dl_i \tag{7}$$

Thus, an n conductor linear system of partial inductance will be

$$\begin{bmatrix} L_{11} & L_{12} & \cdots \\ L_{21} & L_{22} & \cdots \\ \vdots & \vdots & L_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} \int A_1 dl_1 \\ \vdots \\ \int A_n dl_n \end{bmatrix} \tag{8}$$

Therefore, with Eq. (1) and Eq. (8), the partial reluctance matrix will be in the following linear equations,

$$\begin{bmatrix} K_{11} & K_{12} & \cdots \\ K_{21} & K_{22} & \cdots \\ \vdots & \vdots & K_{nn} \end{bmatrix} \begin{bmatrix} \int A_1 dl_1 \\ \vdots \\ \int A_n dl_n \end{bmatrix} = \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} \tag{9}$$

From Eq. (9), we know that if we set the magnetic vector potential drop along conductor segment i to 1 and others to 0, the current on the conductor segments equals the ith column of the partial reluctance matrix K. This gives us another definition of K matrix [8]: "The element K_{ij} is the current flowing through the ith conductor when the magnetic vector potential drop along all conductors, except the jth, are set to zero, and the magnetic vector potential drop along the jth conductor is raised to unit potential." This definition gives us the physical meaning of partial reluctance and makes it possible to extract partial reluctance directly instead of inverse the partial inductance matrix L.

The approach for directly extracting K element is to put unit vector potential drop along the aggressive conductor i and let vector potential drop be zero on other conductors when we want to extract the ith column of K matrix. I.e. let

$$\begin{array}{l} \int A_i dl_i = 1 \\ \int A_j dl_j = 0, j \neq i \end{array} \tag{10}$$

Then based on Eq. (9), K_{ji} equals I_j, the current on conductor segment j. So the target to extract partial reluctance becomes calculating the current distribution.

Here, we should clarify some concept about reluctance and partial reluctance. *Partial inductance* is different from *inductance*, which is a property of closed loops. Similarly, *partial reluctance* is not *reluctance*. If we say *inductance* is a flux controlled non-linear function of flux vs. current, *reluctance* is a current controlled non-linear function of flux vs. current. *Linear reluctance* is the inverse of *linear inductance* and K element is *partial reluctance*. Similar relationship can be found between *capacitance* and *elastance*: *capacitance* is a charge controlled non-linear function of charge vs. voltage and *elastance* is a voltage controlled non-linear function of charge vs. voltage.

III. Locality Of Partial Reluctance

Although it avoids unknown current return loop problem, partial inductance makes the inductance effect global. As it is well known, the loop inductance is not global because the current actually returns in nearby wires for on-chip interconnects [1]. This shows that the inductance effect, or, magnetic interactions of on-chip interconnects should be local in nature and it is the artifical assumptions that cause the on-chip inductance effect global.

Partial inductance extraction under PEEC model [13] shows the reason that the mutual partial inductance decays slowly. As in Fig. 1, during partial inductance extraction, we apply unit current on the aggressive conductor by the unit current source at infinity, and force the current to be zero by applying zero current sources on the victim conductors. Since we force the current only on the aggressor and its virtual return loop at infinity, the magenitic coupling effect exists in the whole space.

Partial reluctance is a different story in the view of its physical meaning, or direct extraction method. As in Fig. 2, we apply unit vector potential drop on the aggressive conductor

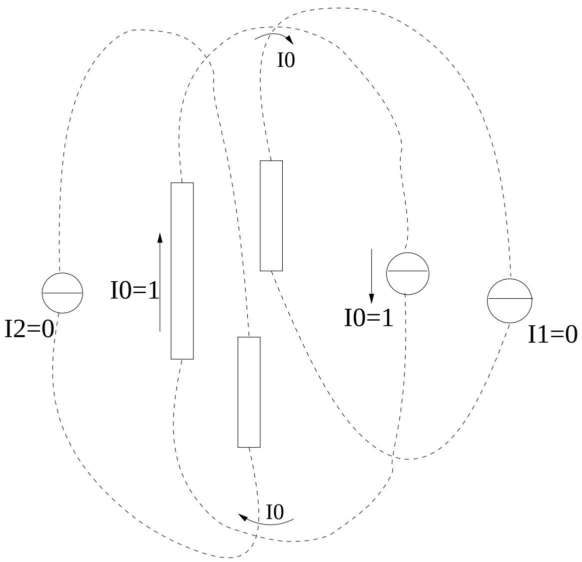

Fig. 1. Partial inductance extraction under PEEC model (The dashed line means the current return loop and the current sources are at infinity.)

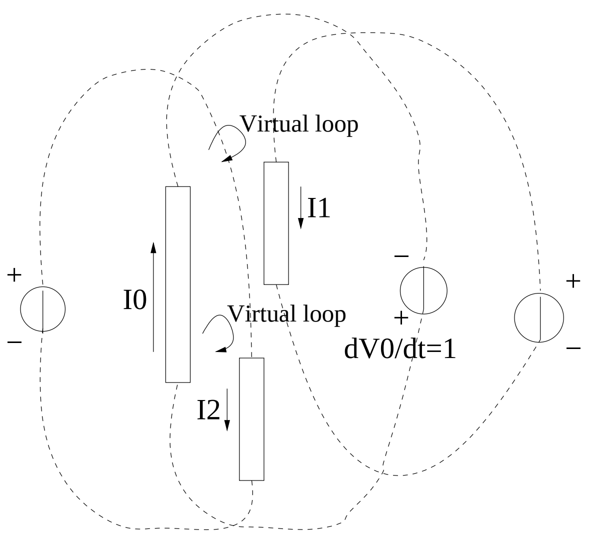

Fig. 2. Partial reluctance extraction by its physical meaning (The dashed line means the current return loop and the voltage sources are at infinity.)

by a voltage source at infinity, and force the vector potential drop on other conductors to be zero by applying negative voltage sources on the victim conductors to cancel the magenitic field induced by the aggressor current. Although the return loop and voltage sources are still at infinity, the currents on the victims in the negative direction actually form the virtual current return loops and thus the faraway magenitic fields are mostly canceled.

The “virtual current return loops” formed naturally when unit or zero flux are forced on the conductors and they need not to be known prior to the extraction. This allows that the K approach has wider application area and better result than other inductance sparsification technics which current return loop need to to assumed first. However, to accelerate partial reluctance extraction (either by traditional L inverse or by direct extraction), some assumptions on current return loop are also needed to use local windowing or similar methods.

IV. Details In Extracting Partial Reluctance

In most cases, the resistance of the conductors is not negligible, which results in more complex partial reluctance extraction.

Since the current is not uniformly distributed at high frequency, we need to mesh the conductors into filaments to simulate the current distribution. With the magneto-quasi-static assumption, we can assume that the current within the conductors flows parallel to their surfaces because there is no charge accumulation on the conductor surfaces. So the the conductor can be divided into a bundle of parallel filaments with rectangular cross-section and the current flows only along the length direction of the filaments.

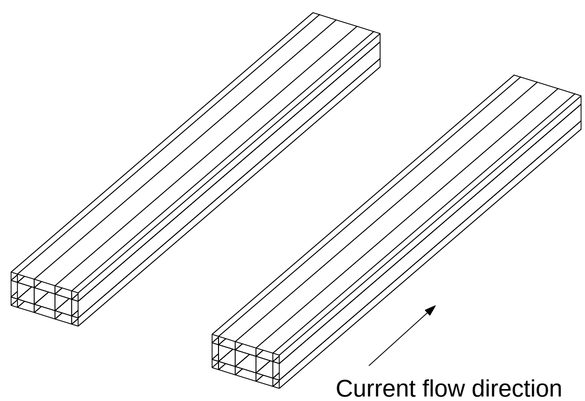

Fig. 3. An example of non-uniformly mesh 2 conductor segments into 2x5x3 filaments

It is well known that the current flows near the surface of the conductor because of skin effect. So we mesh the conductor segments non-uniformly. Fig. 3 gives an example of two parallel conductors been meshed into 5×3 filaments each.

With the sinusoidal steady-state assumption, for a system with n conductors at the frequency of $2\pi\omega$, the current vector $I \in \mathcal{C}^{\backslash}$ and voltage drop vector $V \in \mathcal{C}^{\backslash}$ on the conductors can be expressed as

$$(R + j\omega L)I = V \tag{11}$$

where $R, L \in \mathcal{R}^{\backslash\times\backslash}$ are the resistance matrix and the partial inductance matrix of the system respectively. Transforming Eq. (11) and combining it with the definition of partial reluctance matrix K in Eq. (1), we have

$$j\omega I = K(V - RI) \tag{12}$$

Notice that $V - RI$ is an $n \times 1$ vector and if we set entry i of $V - RI$ to one and the other entries to zero, by computing the resulting current vector I, we can get the ith column of K, which equals to $j\omega I$. Also we can get the scalar potential drop V on each conductor and compute the equivalent resistance.

Since we are more interested in the partial reluctance matrix K rather than the resistance matrix R, we set entry i of $V - RI$ to $j\omega$ and the others to zero. Then the resulting current distribution I equals column i of K.

With the definition of vector potential A in Eq. (2) and Faraday's Law in sinusoidal steady-state, we have

$$E = -j\omega A - \nabla\Phi \tag{13}$$

where Φ is the scalar potential and E is the electronic field. Since the current flow is parallel to the surface of the conductor, we can only consider E, A and $\nabla\Phi$ along the length of the conductor. Integrate both sides of Eq. (13) along the length direction of the conductor from one end a to the other end b, we have

$$V_{ab} = -(\Phi_b - \Phi_a) = j\omega \int_a^b A dl + E \cdot l_{ab} \tag{14}$$

As E contributes to the resistive potential drop from $J = \sigma E$, we have

$$E \cdot l_{ab} = RI \tag{15}$$

Combine Eq. (14) and Eq. (15)

$$V - RI = j\omega \int_a^b A dl \tag{16}$$

So setting the $V - RI$ to $j\omega$ on the aggressive conductor and others to zero is the same as setting the vector potential drop along the aggressive conductor to one and others to zero, which is consistent with the secondary definition of partial reluctance and the approach for extracting partial reluctance we stated in section II.

Assuming each filament is thin enough that the current can be approximated uniformly distributed inside the filament, $\hat{I}_i$ and $\hat{V}_i$ [1], the current and the potential drop on filament i, can be given

$$\hat{R}_{ii}\hat{I}_i + j\omega \sum_{j=1}^{m} \hat{L}_{ij}\hat{I}_j = \hat{V}_i \tag{17}$$

where m is the number of the filaments. The filament's DC resistance $\hat{R}_{ii}$ can be given

$$\hat{R}_{ii} = \frac{l_i}{\sigma \hat{a}_i} \tag{18}$$

where σ is the conductivity of the conductors and the mutual partial inductance between the filaments (or self partial inductance if $i = j$) $\hat{L}_{ij}$ has the following formulation similar to Eq. (6)

$$\hat{L}_{ij} = \frac{\mu_0}{4\pi \hat{a}_i \hat{a}_j}\left[\int_{\hat{a}_i}\int_{\hat{a}_j}\int_{l_i}\int_{l_j} \frac{d\vec{l}_i d\vec{l}_j}{\|\vec{r}_i - \vec{r}_j\|} d\hat{a}_i d\hat{a}_j\right] \tag{19}$$

[1] Here we use a little hat ˆ to distinguish the symbols for filaments from the symbols for conductor segments.

Because uniform distributed current is assumed inside the filaments, Eq. (19) can be accurately integrated by Hoer's formula [7].

Eq. (17) can be written in matrix form

$$(\hat{R} + j\omega\hat{L})\hat{I} = \hat{V} \tag{20}$$

where $\hat{R}$ is an $m \times m$ diagonal matrix and $\hat{L}$ is an $m \times m$ matrix. They are known when the conductors are meshed into filaments. $\hat{I} \in \mathcal{C}^{\Updownarrow}$ is the vector of current on the filaments, which is what we want to know and $\hat{V} \in \mathcal{C}^{\Updownarrow}$ is the vector of potential drop on the m filaments.

Because the transverse current is zero, the potential drop on different filaments of a same conductor are equal. By defining the mesh incidence matrix $M \in \mathcal{R}^{\setminus\times\Updownarrow}$ as $M_{ij} = 1$ when filament j is in conductor i and $M_{ji} = 0$ otherwise, we have

$$\hat{V} = M^t V \tag{21}$$

and

$$I = M\hat{I} \tag{22}$$

Since the partial reluctance matrix K and resistance matrix R are real, the resulting current I must be real when the vector potential drop $\int Adl_i$ is set to 1 on the aggressor and 0 on the victim. I.e.

$$I_{im} = 0 \tag{23}$$

Also the real part of potential drop V contributes the potential drop caused by the electric field E and its image part contributes the potential drop caused by the magnetic field $\int Adl$, which are set to 1 or 0 for computing K matrix. Then Eq. (20) can be separated into two equations

$$\begin{cases} \hat{R}\hat{I}_{re} - \omega\hat{L}\hat{I}_{im} = \hat{V}_{re} = M^t V_{re} \\ \omega\hat{L}\hat{I}_{re} + \hat{R}\hat{I}_{im} = \hat{V}_{im} = \omega M^t \int Adl \end{cases} \tag{24}$$

where $\hat{I} = \hat{I}_{re} + j\hat{I}_{im}$, $\hat{V} = \hat{V}_{re} + j\hat{V}_{im}$ and $V = V_{re} + jV_{im} = V_{re} + j\omega \int Adl$.

Eq. (24) and Eq. (23) can be put together in matrix form

$$\begin{bmatrix} \hat{R} & -\omega\hat{L} & -M^t \\ \omega\hat{L} & \hat{R} & 0 \\ 0 & M & 0 \end{bmatrix} \begin{bmatrix} \hat{I}_{re} \\ \hat{I}_{im} \\ V_{re} \end{bmatrix} = \begin{bmatrix} 0 \\ \omega M^t \int Adl \\ 0 \end{bmatrix} \tag{25}$$

Then we will have $\hat{I}_{re}$, $\hat{I}_{im}$ and $\hat{V}_{re}$ by solving linear Eq. (25) and get the current on the conductor segments $I = M\hat{I}$. The ith column of partial reluctance matrix K equals I and the equivalent resistance of conductor segment i at the frequency of $2\pi\omega$ is given by $R_{ii} = \mathcal{R}\rceil(\mathcal{V}_\rangle)/\mathcal{I}_\rangle$.

The mutual partial inductance of two filaments can be got by a simple closed formula under the condition that the current flow is uniform inside each conductor filament. This is because other conductors in the space have no effect on the mutual partial inductance between the two filaments. However, other conductors do have effect on the mutual partial reluctance between two filaments. Since then, it is impossible to find a simple direct formula for the mutual partial reluctance of