Directly Extracting On-Chip Circuit Element Partial Reluctance K

Yu Du, Hao Ji, Wayne Dai
School of Engineering
UC Santa Cruz
Santa Cruz, CA95064
duyu,hji,dai@soe.ucsc.edu

Aprial 15, 2003

TR number: UCSC-CRL-0303

Abstract

This paper provides a method to directly extract K element, or *partial reluctance*. Partial reluctance K is a new circuit element recently introduced [1] to capture the on-chip inductance effect. We use a 3-D discretization of a magneto-quasi-static integral formulation to simulate the current distribution and get the frequency dependent partial reluctance matrix K and resistance R. Directly extracting partial reluctance gives a clearer view of partial reluctance's physical meaning and avoids inverting the partial inductance matrix before running simulation and reduction based on RKC model. Also, benefiting from the locality of the partial reluctance, a hierarchical shielding technique is used in the extraction and it brings a great speed up. The result from examples shows advantages in direct partial reluctance extraction with shielding techniques over the traditional partial reluctance extraction by inverting L.

keywords: **Partial Reluctance, Partial Inductance, Shielding, Direct Extraction**

1 Introduction

As clock speed increases and less resistive wires are used to enhance on-chip signal propagation speed, inductance effect of on-chip interconnect is becoming more and more important. However, capturing on-chip inductance effect is very difficult because of unknown circuit return path. Ruehli introduced the concept of partial inductance developed by Rosa [11] to the circuit design field [12] to avoid the unknown loop problem. But since the partial inductance is based on the virtual loop closed at infinity, the coupling are now among all the conductor segments so that the resulting partial inductance matrix is extremely dense. Due to this global effect of dense partial inductance matrix, people are facing many difficulties on extracting on-chip parasitic parameter and simulating on-chip interconnect with RLC model.

Many work had been done to make the partial inductance sparse, such as the "shift-truncate" method [10] and "return-limited loop inductance" concept [13]. However, the accuracy of these approaches are not guaranteed under different interconnect topology.

Fortunately, the inverse of partial inductance matrix, the *partial reluctance* matrix, has locality and stability [1]. The new circuit element, the partial reluctance, or K element, was first introduced by Devgan *et al* [1]. Ji *et al* showed the locality and positive definiteness of partial reluctance in [7] and developed a new circuit simulation tool, *KSim*, by incorporating the K element. Since the locality and stability of partial reluctance matrix K, KSim can achieve much higher performance than today's circuit simulation tools based on RLC model.

Other approaches based on the concept of partial reluctance such as "double inversion" by Beattie *et al* in [2] and "inductwise" by Chen *et al* in [4] are also proposed and great advantages over RLC model are shown in their work. And Zheng *et al* had done some work on model order reduction for RKC model in [14].

The previous approaches [4] [7] to extract partial reluctance are to divide the circuit into some smaller parts, get the partial inductance matrices L of each one from available extraction tool such as FastHenry [9], inverse the L matrices to the K matrices and combine the small K matrices to a big one for the input of RKC simulation tools. It is clear that the matrix inversion is costly, and developing extraction method for directly catching partial reluctance becomes necessary.

Also, the partial reluctance shares many features with capacitance because one uni-directional magnetic problem can be transformed in to an electric problem [4]. So direct partial reluctance extraction may utilize some techniques used in capacitance extraction and gain the advantage over inductance extraction. This paper uses the hierarchical shielding algorithm, which was similar to the technique used in capacitance extraction [8], to accelerate the partial reluctance extraction.

The following section will give an overview about the background of partial reluctance. Section 3 discusses some details of partial reluctance direct extraction. Hierarchical shielding algorithm to accelerate partial reluctance extraction is presented in section 4 and the experimental results are given in section 5 to show the advantage of direct K extraction. Finally, section 6 concludes this paper.

2 Physical Background of Partial Reluctance

In [1], partial reluctance matrix K is defined as the inverse of partial inductance matrix L.

$$[K] = [L]^{-1} \tag{1}$$

The magnetic vector potential, A, is defined as

$$\begin{array}{c} \nabla \times A = \mu H \\ \nabla \cdot A = 0 \end{array} \tag{2}$$

Then by magneto-quasistatic approximation [5] and sinusoidal steady-state assumption, the vector potential has the following relation to the current distribution $J(r)$

$$A(r) = \frac{\mu_0}{4\pi} \int_{V'} \frac{J(r')}{\|\vec{r}_i - \vec{r}_j\|} dv' \tag{3}$$

where V' is the volume of all conductors. Then A_{ij}, the vector potential on conductor segment i due to the current on conductor segment j is

$$A_{ij} = \frac{\mu_0}{4\pi a_j} \int_{a_j} \int_{l_j} \frac{I_j d\vec{l_j}}{\|\vec{r}_i - \vec{r}_j\|} da_j \tag{4}$$

where l_j and a_j are the length and the cross section area of filament j. I_j is the current on conductor segment j. And A_i, the vector potential on conductor segment i due to all the current in the system, is

$$A_i = \sum_{j=1}^{n} A_{ij} \tag{5}$$

Also we know that with magneto-quasistatic approximation, the partial inductance between two conductor segments i and j is as follows [12]

$$L_{ij} = \frac{\mu_0}{4\pi a_i a_j} [\int_{a_i} \int_{a_j} \int_{l_i} \int_{l_j} \frac{d\vec{l_i} d\vec{l_j}}{\|\vec{r}_i - \vec{r}_j\|} da_i da_j] \tag{6}$$

With the definition of magnetic vector potential in Eq. (2) and zero transverse current on the conductors, A_i distributes uniformly on the cross-section of conductor segment i. So the magnetic vector potential along a conductor segment can be simply wrote down as

$$\frac{1}{a_i} \int \int A_i da_i dl_i = \int A_i dl_i \tag{7}$$

Thus, an n conductor linear system of partial inductance will be

$$\begin{bmatrix} L_{11} & L_{12} & \cdots \\ L_{21} & L_{22} & \cdots \\ \vdots & \vdots & L_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} \int A_1 dl_1 \\ \vdots \\ \int A_n dl_n \end{bmatrix} \tag{8}$$

Therefore, with Eq. (1) and Eq. (8), the partial reluctance matrix will be in the following linear equations,

$$\begin{bmatrix} K_{11} & K_{12} & \cdots \\ K_{21} & K_{22} & \cdots \\ \vdots & \vdots & K_{nn} \end{bmatrix} \begin{bmatrix} \int A_1 dl_1 \\ \vdots \\ \int A_n dl_n \end{bmatrix} = \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} \tag{9}$$

From Eq. (9), we know that if we set the magnetic vector potential drop along conductor segment i to 1 and others to 0, the current on the conductor segments equals the ith column of the partial reluctance matrix K. This gives us another definition of K matrix [7]: "The element K_{ij} is the current flowing through the ith conductor when the magnetic vector potential drop along all conductors, except the jth, are set to zero, and the magnetic vector potential drop along the jth conductor is raised to unit potential." This definition gives us the physical meaning of partial reluctance and makes it possible to extract partial reluctance directly instead of inverse the partial inductance matrix L.

The approach for directly extracting K element is to put unit vector potential drop along the aggressive conductor i and let vector potential drop be zero on other conductors when we want to extract the ith column of K matrix. I.e. let

$$\begin{aligned} &\int A_i dl_i = 1 \\ &\int A_j dl_j = 0, j \neq i \end{aligned} \tag{10}$$

Then based on Eq. (9), K_{ji} equals I_j, the current on conductor segment j. So the target to extract partial reluctance becomes calculating the current distribution.

Here, we should clarify some concept about reluctance and partial reluctance. *Partial inductance* is different from *inductance*, which is a property of closed loops. Similarly, *partial reluctance* is not *reluctance*. If we say *inductance* is a flux controlled non-linear function of flux vs. current, *reluctance* is a current controlled non-linear function of flux vs. current. *Linear reluctance* is the inverse of *linear inductance* and K element is *partial reluctance*. Similar relationship can be found between *capacitance* and *elastance*: *capacitance* is a charge controlled non-linear function of charge vs. voltage and *elastance* is a voltage controlled non-linear function of charge vs. voltage.

3 Extracting Partial Reluctance

In most cases, the resistance of the conductors is not negligible, which results in more complex partial reluctance extraction.

Since the current is not uniformly distributed at high frequency, we need to mesh the conductors into filaments to simulate the current distribution. With the magneto-quasi-static assumption, we can assume that the current within the conductors flows parallel to their surfaces because there is no charge accumulation on the conductor surfaces. So the the conductor can be divided into a bundle of parallel filaments with rectangular cross-section and the current flows only along the length direction of the filaments.

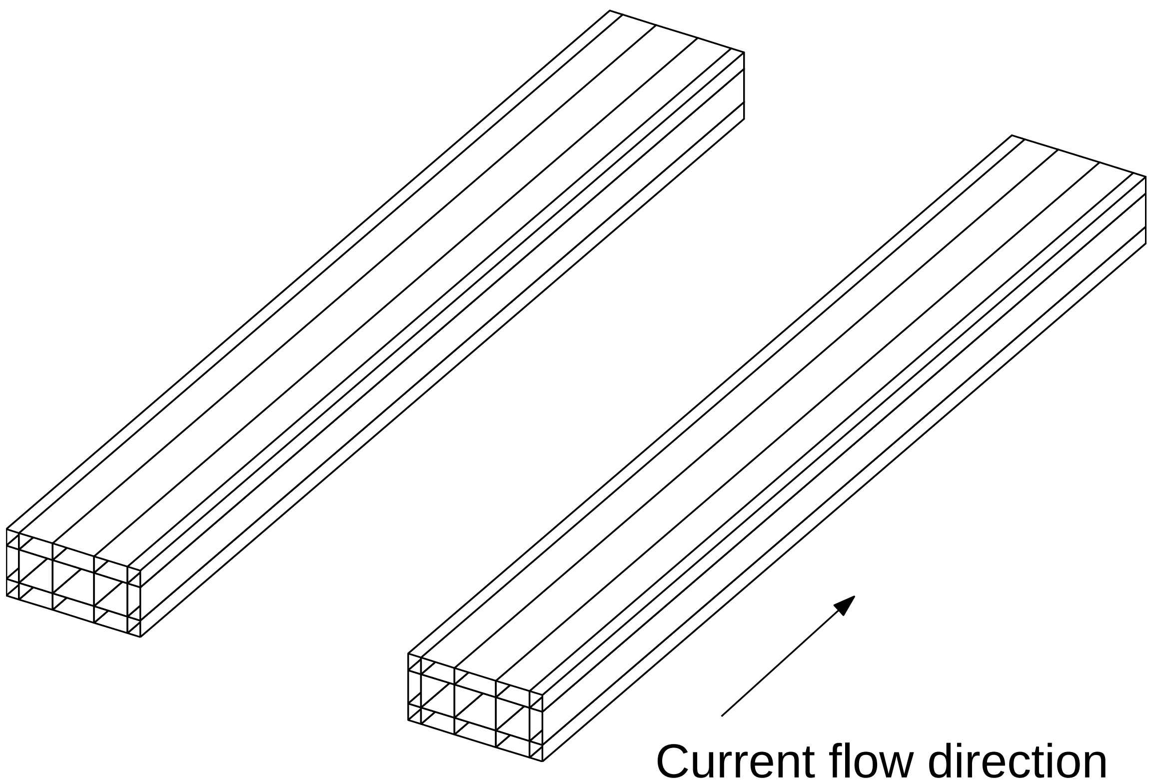

Figure 1: An example of non-uniformly mesh 2 conductor segments into 2x5x3 filaments

It is well known that the current flows near the surface of the conductor because of skin effect. So we mesh the conductor segments non-uniformly. Fig. 1 gives an example of two parallel conductors been meshed into 5×3 filaments each.

With the sinusoidal steady-state assumption, for a system with n conductors at the frequency of $2\pi\omega$, the current vector $I \in \mathcal{C}^n$ and voltage drop vector $V \in \mathcal{C}^n$ on the conductors can be expressed as

$$(R + j\omega L)I = V \tag{11}$$

where $R, L \in \mathcal{R}^{n\times n}$ are the resistance matrix and the partial inductance matrix of the system respectively. Transforming Eq. (11) and combining it with the definition of partial reluctance matrix K in Eq. (1), we have

$$j\omega I = K(V - RI) \tag{12}$$

Notice that $V - RI$ is an $n \times 1$ vector and if we set entry i of $V - RI$ to one and the other entries to zero, by computing the resulting current vector I, we can get the ith column of K, which equals to $j\omega I$. Also we can get the scalar potential drop V on each conductor and compute the equivalent resistance.

Since we are more interested in the partial reluctance matrix K rather than the resistance matrix R, we set entry i of $V - RI$ to $j\omega$ and the others to zero. Then the resulting current distribution I equals column i of K.

With the definition of vector potential A in Eq. (2) and Faraday's Law in sinusoidal steady-state, we have

$$E = -j\omega A - \nabla\Phi \tag{13}$$

where Φ is the scalar potential and E is the electronic field. Since the current flow is parallel to the surface of the conductor, we can only consider E, A and $\nabla\Phi$ along the length of the conductor. Integrate both sides of Eq. (13) along the length direction of the conductor from one end a to the other end b, we have

$$V_{ab} = -(\Phi_b - \Phi_a) = j\omega \int_a^b A dl + E \cdot l_{ab} \tag{14}$$

As E contributes to the resistive potential drop from $J = \sigma E$, we have

$$E \cdot l_{ab} = RI \tag{15}$$

Combine Eq. (14) and Eq. (15)

$$V - RI = j\omega \int_a^b A dl \tag{16}$$

So setting the $V - RI$ to $j\omega$ on the aggressive conductor and others to zero is the same as setting the vector potential drop along the aggressive conductor to one and others to zero, which is consistent with the secondary definition of partial reluctance and the approach for extracting partial reluctance we stated in section 2.

Assuming each filament is thin enough that the current can be approximated uniformly distributed inside the filament, $\hat{I}_i$ and $\hat{V}_i$ [1], the current and the potential drop on filament i, can be given

$$\hat{R}_{ii}\hat{I}_i + j\omega \sum_{j=1}^{m} \hat{L}_{ij}\hat{I}_j = \hat{V}_i \tag{17}$$

where m is the number of the filaments. The filament's DC resistance $\hat{R}_{ii}$ can be given

$$\hat{R}_{ii} = \frac{l_i}{\sigma \hat{a}_i} \tag{18}$$

where σ is the conductivity of the conductors and the mutual partial inductance between the filaments (or self partial inductance if $i = j$) $\hat{L}_{ij}$ has the following formulation similar to Eq. (6)

$$\hat{L}_{ij} = \frac{\mu_0}{4\pi \hat{a}_i \hat{a}_j} [\int_{\hat{a}_i} \int_{\hat{a}_j} \int_{l_i} \int_{l_j} \frac{d\vec{l_i} d\vec{l_j}}{\|\vec{r}_i - \vec{r}_j\|} d\hat{a}_i d\hat{a}_j] \tag{19}$$

Because uniform distributed current is assumed inside the filaments, Eq. (19) can be accurately integrated by Hoer's formula [6].

Eq. (17) can be written in matrix form

$$(\hat{R} + j\omega\hat{L})\hat{I} = \hat{V} \tag{20}$$

where $\hat{R}$ is an $m \times m$ diagonal matrix and $\hat{L}$ is an $m \times m$ matrix. They are known when the conductors are meshed into filaments. $\hat{I} \in \mathcal{C}^{\Updownarrow}$ is the vector of current on the filaments, which is what we want to know and $\hat{V} \in \mathcal{C}^{\Updownarrow}$ is the vector of potential drop on the m filaments.

Because the transverse current is zero, the potential drop on different filaments of a same conductor are equal. By defining the mesh incidence matrix $M \in \mathcal{R}^{\Updownarrow \times \backslash}$ as $M_{ji} = 1$ when filament j is in conductor i and $M_{ji} = 0$ otherwise, we have

$$\hat{V} = M^t V \tag{21}$$

[1] Here we use a little hat ˆ to distinguish the symbols for filaments from the symbols for conductor segments.

and

$$I = M\hat{I} \tag{22}$$

Since the partial reluctance matrix K and resistance matrix R are real, the resulting current I must be real when the vector potential drop $\int Adl_i$ is set to 1 on the aggressor and 0 on the victim. I.e.

$$I_{im} = 0 \tag{23}$$

Also the real part of potential drop V contributes the potential drop caused by the electric field E and its image part contributes the potential drop caused by the magnetic field $\int Adl$, which are set to 1 or 0 for computing K matrix. Then Eq. (20) can be separated into two equations

$$\begin{cases} \hat{R}\hat{I}_{re} - \omega\hat{L}\hat{I}_{im} = \hat{V}_{re} = M^t V_{re} \\ \omega\hat{L}\hat{I}_{re} + \hat{R}\hat{I}_{im} = \hat{V}_{im} = \omega M^t \int Adl \end{cases} \tag{24}$$

where $\hat{I} = \hat{I}_{re} + j\hat{I}_{im}$, $\hat{V} = \hat{V}_{re} + j\hat{V}_{im}$ and $V = V_{re} + jV_{im} = V_{re} + j\omega \int Adl$.

Eq. (24) and Eq. (23) can be put together in matrix form

$$\begin{bmatrix} \hat{R} & -\omega\hat{L} & -M^t \\ \omega\hat{L} & \hat{R} & 0 \\ 0 & M & 0 \end{bmatrix} \begin{bmatrix} \hat{I}_{re} \\ \hat{I}_{im} \\ V_{re} \end{bmatrix} = \begin{bmatrix} 0 \\ \omega M^t \int Adl \\ 0 \end{bmatrix} \tag{25}$$

Then we will have $\hat{I}_{re}$, $\hat{I}_{im}$ and $\hat{V}_{re}$ by solving linear Eq. (25) and get the current on the conductor segments $I = M\hat{I}$. The ith column of partial reluctance matrix K equals I and the equivalent resistance of conductor segment i at the frequency of $2\pi\omega$ is given by $R_{ii} = \mathcal{R}](\mathcal{V}_\rangle)/\mathcal{I}_\rangle$.

The mutual partial inductance of two filaments can be got by a simple closed formula under the condition that the current flow is uniform inside each conductor filament. This is because other conductors in the space have no effect on the mutual partial inductance between the two filaments. However, other conductors do have effect on the mutual partial reluctance between two filaments. Since then, it is impossible to find a simple direct formula for the mutual partial reluctance of two filaments only with the information of the two filaments. So it still needs to solve equations of the whole system to get the partial reluctance directly.

Here is a small example to show a conductor's impact on mutual inductance and reluctance between other conductors. As in Fig. 2, the structure is a bus structure with four equal length parallel conductors.

We have the partial inductance matrix of the bus structure

$$L = \begin{bmatrix} 2.79 & 1.48 & 1.05 & 0.82 \\ 1.48 & 2.76 & 1.47 & 1.05 \\ 1.05 & 1.47 & 2.76 & 1.48 \\ 0.82 & 1.05 & 1.48 & 2.79 \end{bmatrix} \times 10^{-11} H \tag{26}$$

and the partial reluctance matrix

$$K = \begin{bmatrix} 5.13 & -2.36 & -0.51 & -0.35 \\ -2.36 & 6.23 & -2.16 & -0.51 \\ -0.51 & -2.16 & 6.24 & -2.36 \\ -0.35 & -0.51 & -2.35 & 5.13 \end{bmatrix} \times 10^{10} H^{-1} \tag{27}$$

Remove conductor 2 and conductor 3 in the bus structure (marked in dash lines in Fig. 2) and the inductance and reluctance matrix will be as follows,

$$L' = \begin{bmatrix} 2.79 & 0.82 \\ 0.82 & 2.79 \end{bmatrix} \times 10^{-11} H \tag{28}$$

$$K' = \begin{bmatrix} 3.92 & -1.15 \\ -1.15 & 3.92 \end{bmatrix} \times 10^{10} H^{-1} \tag{29}$$

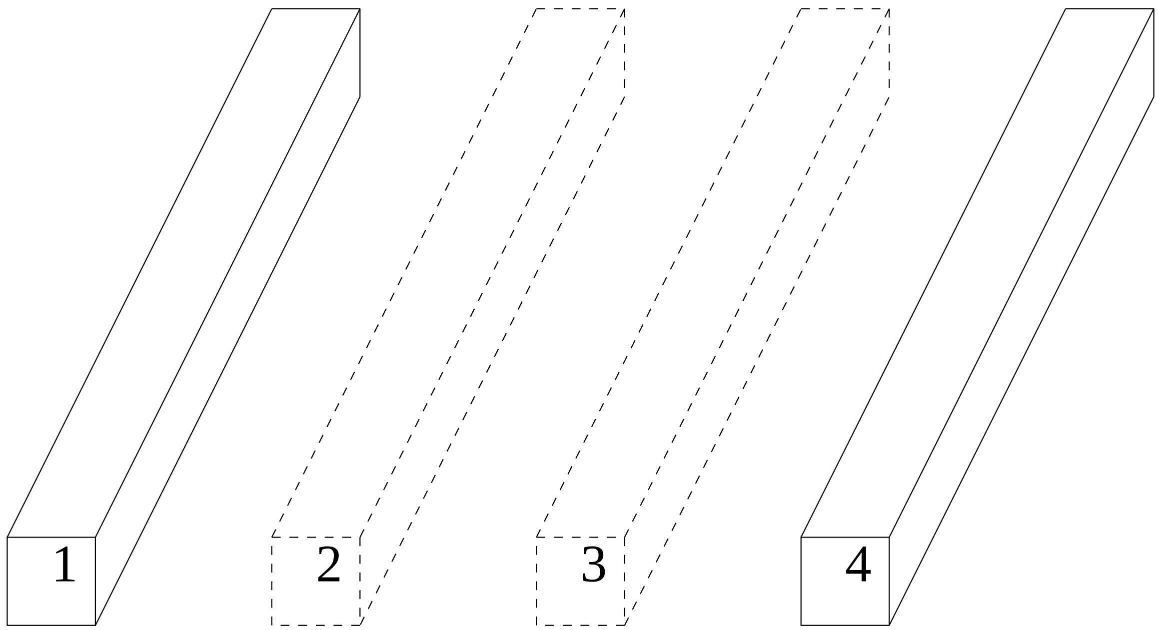

Figure 2: A four equal length parallel conductor bus structure

The mutual partial inductance of conductor 1 and conductor 4 is $L_{14} = 0.82 \times 10^{-11}H$ in Eq. (26) and $L'_{12} = 0.82 \times 10^{-11}H$ in Eq. (28) after remove conductor 2 and 3. This shows that conductor 2 and conductor 3 has no effect on the mutual inductance between conductor 1 and 4. But the mutual reluctance is different. $K_{14} = -0.35 \times 10^{10}H^{-1}$ in Eq. (27) and $K'_{12} = -1.15 \times 10^{10}H^{-1}$, which is 3 times smaller with the effect of conductor 2 and 3.

It is also needed to simulate the current distribution and solve the whole system when extracting partial inductance because the uniform distribution assumption does not hold in real world. So extracting partial reluctance will not consume more time than partial inductance extracting. And benefiting from the locality of partial reluctance, direct reluctance extracting yields much shorter computing time.

The self and mutual inductance of the filaments are used in calculating the partial reluctance of the conductors. But this is not the same as partial reluctance extraction by inverting the partial inductance matrices. The L matrix to be inverted for K matrix is the partial inductance of conductors while the inductance matrix $\hat{L}$ in direct reluctance extraction is inductance of filaments.

4 Hierarchical Shielding Approach

Partial reluctance has some capacitance-like features such as locality and stability [1]. Although these features have not been proved strictly yet, Ji *et al* illustrated them intuitively in [7] and later, Chen *et at* stated that "every uni-directional magnetic problem can be transformed into an electric one" in [4].

Capacitance has locality because there is shielding effect in electric field. As Fig. 3 shows, conductor 5 is shielded by conductor 3 in conductor 1's view. When conductor 1 is charged as the aggressor, the charge on conductor 5 is nearly zero so that C_{15}, the capacitance between conductor 1 and 5, can be neglected.

Taking the advantage of capacitance's locality, some fast interconnect capacitance extraction tool such as Gu *et al* [8] discards shielded conductors to accelerate the extraction. These acceleration techniques cannot be used on interconnect inductance extraction because partial inductance effect is global. But partial reluctance has locality and conductor shielding methods can be used.

Fig. 4 shows the shielding effect of partial reluctance on three parallel conductors. Conductor 1 is the aggressive conductor while conductor 2 and 3 are the victims. The magnetic field incited by I_1, the current

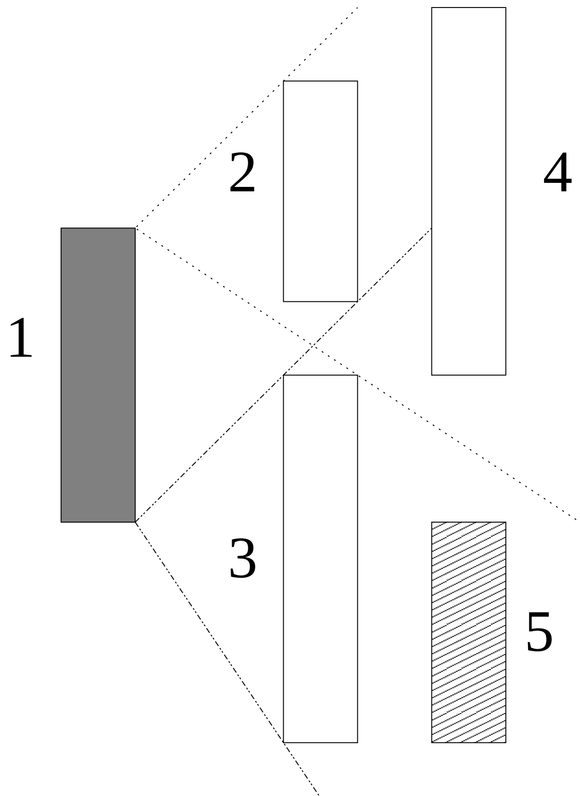

Figure 3: The shielding effect of interconnect capacitance

on conductor 1, and the magnetic field incited by I_2 canceled each other at the space around conductor 3. So I_3, the incited current on conductor 3, is very small.

Chen *et at* proposed Window Selection Algorithm in [4] to discard shielded conductors before inductance/reluctance extraction. But it can be noticed that the magnetic field cannot be completely shielded. So discarding all the shielded conductor segments (shielding level 1 in [4]) may result in big error. To get higher accuracy, WS algorithm increases the level of shielding, which means discarding the conductors only shielded by more levels of shielding conductors. But higher level of shielding increases the extraction time substantially.

Directly partial reluctance extraction has much more flexibility in dealing with shielded and partially shielded conductors. We proposed a *hierarchical shielding* algorithm to accelerate the partial reluctance extraction. The level of shielding are determined as follows,

- The aggressive conductor is defined as level 0, the highest level.
- Extends both ends of a level i conductor by a factor of x. A conductor is defined in level $i+1$ if
  - It is not in any level j where $j < i$
  - It is in the area swept in the direction perpendicular to the current flow direction (Fig. 5)
  - There are no conductor shielding between it and the level i conductor

Fig. 5 shows an example of the conductors in different levels. Conductor 2 is the aggressor and it is in level 0; conductor 3 and 5 is in level 1; conductor 1, 4 and 7 is in level 2 and conductor 6 is in level 3.

Fig. 6 shows the current on 5 parallel conductors when conductor 1 is the aggressor and the unit vector potential drop is applied on it. The current on conductor 4 and 5 is very small because of the shielding effect of partial reluctance. But the current on conductor 3 is not small enough to be neglected. So if we simply discard all the shielded conductors, the result maybe not accurate enough.

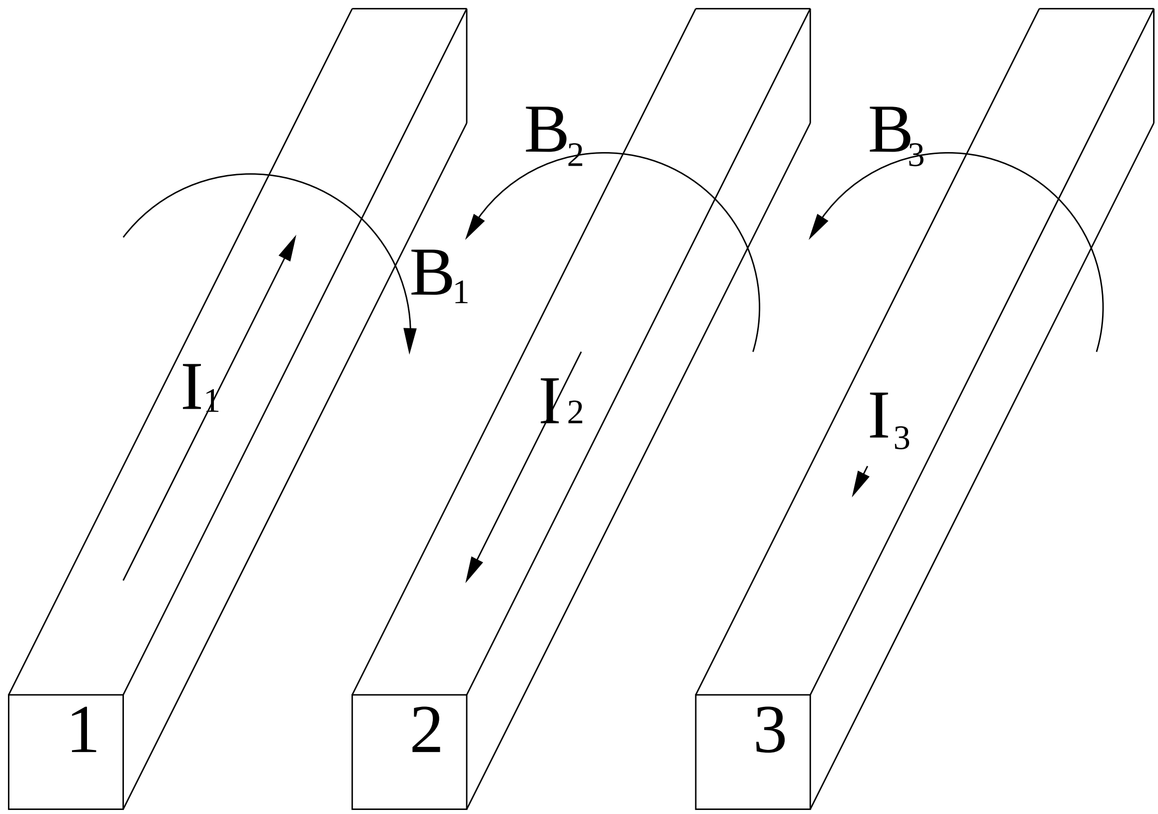

Figure 4: The shielding effect of interconnect partial reluctance

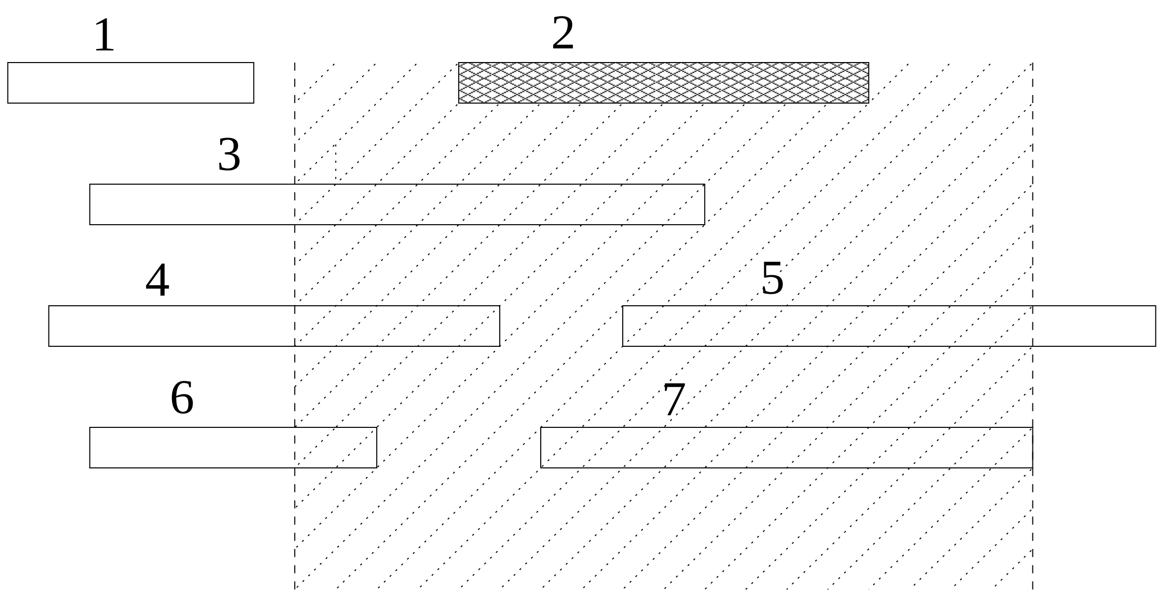

Figure 5: Conductors of different shielding level

In hierarchical shielding algorithm, the different level of conductors are treated in different ways. Fine mesh are used on higher level of conductors, such as level 0 and level 1 conductors. Less fine mesh are used on lower level of conductors because the current on them is not significant to the whole system. And the

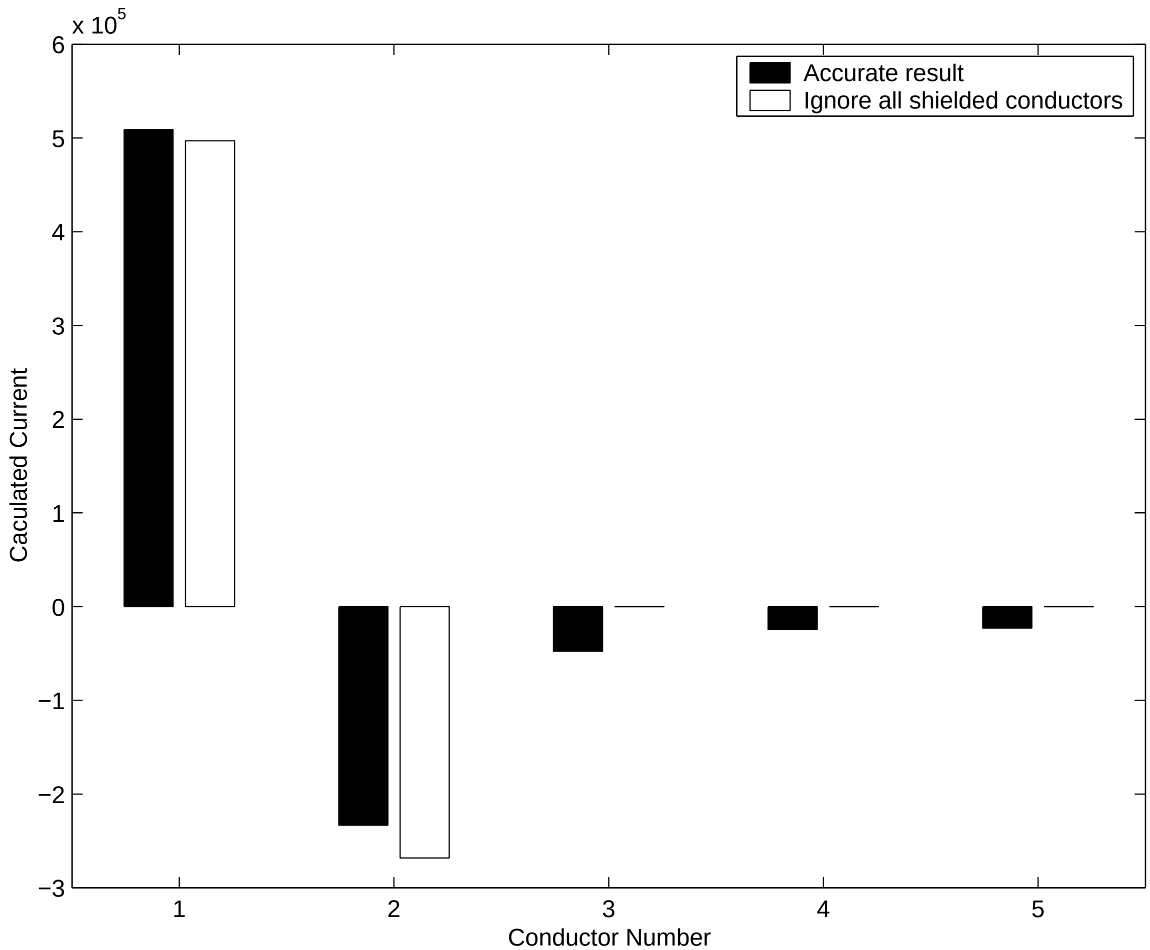

Figure 6: The current on the parallel conductors

conductors with level lower than a certain threshold will be simply discarded. In this way, the contribution of the shielded conductors can be captured without lower the extraction speed too much.

Fig. 7 shows the current on the same 5 parallel conductors and conductor 1 is the aggressor. Hierarchical shielding algorithm is used in the current simulation to compare with the accurate calculation in which all the conductors are finely meshed. We can see that the result of hierarchical shielding algorithm is very near to the accurate result.

Since conductors does not have shielding effect on mutual partial inductance, hierarchical shielding algorithm cannot be applied on traditional partial reluctance extraction from partial inductance matrices inversion. The mutual inductance between shielded conductors is still very large and only coarse mesh on the victim conductor can not capture the skin effect of current distribution well enough. The experimental results shown in section 5 will give a clearer idea about it.

Because shielding algorithms are used in direct partial reluctance extraction, the size of the linear system we need to solve is not large regardless the total number of conductors. With the relatively small number of non-zeros in the matrix, LU decomposition is enough for solve the linear equations.

5 Experimental Results

We developed our extraction tool directly extracting partial reluctance in C++ programming language. Simple shielding and hierarchical shielding algorithm are implemented in the extraction program. The results

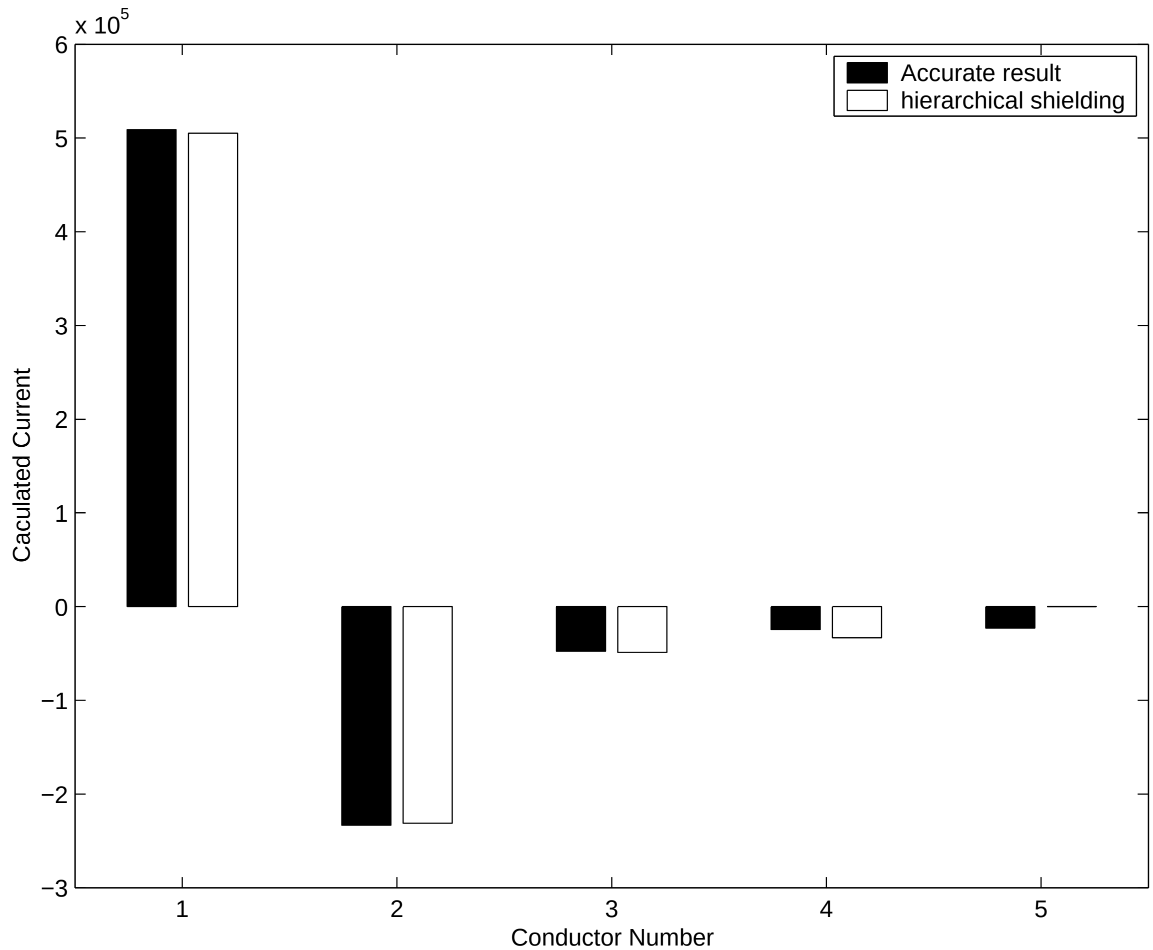

Figure 7: The current calculation by the hierarchical shielding algorithm

are compared with traditional inductance extraction, FastHenry [9] and partial reluctance extraction by inverting small inductance matrices. Our extraction and simulation program was run on a Sun UltraSPARC-II 296MHz system.

A random generated circuit with 300 ports is used as a testing example. The conductors in the circuit have unequal lengths and are placed parallel. The conductivity is $2.0 \times 10^7 \Omega^{-1} \cdot m^{-1}$ and the frequency is 1.0×10^8 Hz. We use FastHenry to extract the full circuit to get the full partial inductance matrix. The inversion of full partial inductance matrix is viewed as standard partial reluctance matrix and SPICE simulation with full partial inductance matrix is viewed as accurate simulation result. In extracting this 300-port circuit, 1 level, 3 level windowing and hierarchical shielding are applied both on direct K extraction and K extraction by inverting small L matrices.

Fig. 8 shows the accuracy of extracting partial reluctance in different ways. Here we use the eigenvalues of the partial reluctance matrices to illustrate the accuracy of different methods.

Fig. 8 shows that the result got by direct K extraction with hierarchical shielding and 3 level windowing are both very near to the inverting of full partial inductance matrix from FastHenry, which is regarded as accurate result. But there is some visible error of the partial reluctance got by 1 level windowing and inductance inverting with hierarchical shielding. This shows that we cannot get the result accurate enough when too many conductors are discarded and hierarchical shielding can only be applied on direct reluctance extraction.

Extraction method	Extracting time	Simulation time	1st peak error	1st drop error
Full inductance extraction by FastHenry	18087.2(sec)	127.8(sec)	-	-
Inductance inverse with 1 level windowing	7.44(sec)	2.93(sec)	18.0%	85.7%
Inductance inverse with 3 level windowing	188.06(sec)	8.76(sec)	1.41%	2.45%
Inductance inverse with hierarchical shielding	15.92(sec)	6.49(sec)	12.3%	58.5%
Direct K with 1 level windowing	6.46(sec)	2.93(sec)	17.7%	85.1%
Direct K with 3 level windowing	155.46(sec)	8.76(sec)	1.43%	2.46%
Direct K with hierarchical shielding	11.19(sec)	6.51(sec)	1.92%	2.05%

Table 1: Speed and accuracy comparison of different approaches

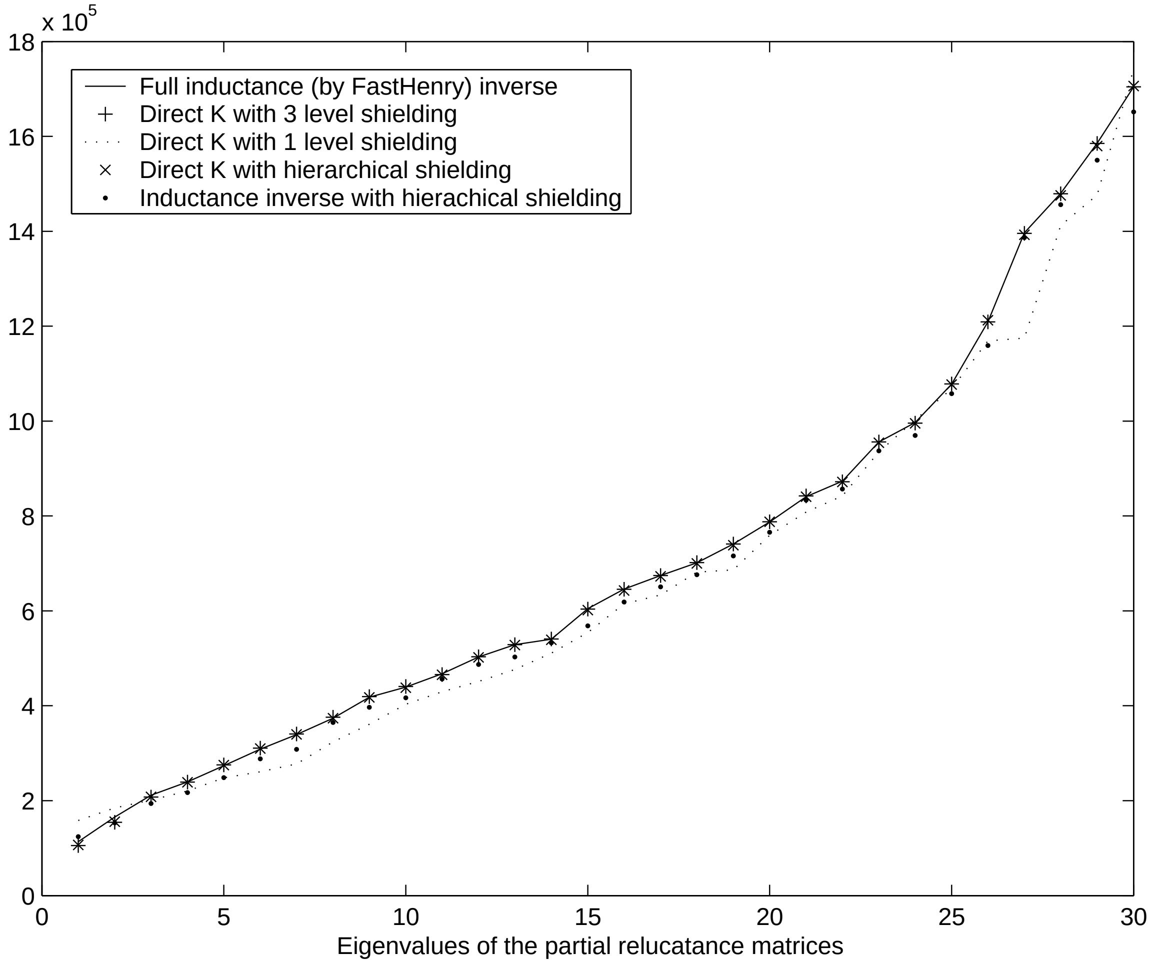

Figure 8: The eigenvalues of the partial reluctance matrices got from different ways

We put the partial reluctance matrices got by different methods into INDUCTWISE [4][3] [2] to simulate the circuit. And compare these K simulation result with SPICE simulation, which uses full inductance matrix from FastHenry as input. In the same 300-port circuit, we activate one conductor with a current step function and observe the response voltage waveforms on a nearby victim conductor. Fig. 9 shows the waveforms on the victim conductor using different extraction results for INDUCTWISE input. And Tab. 1 shows the extraction time and simulation accuracy in these different extraction methods.

Tab. 1 shows that direct extracting methods with shielding acceleration can run much faster than ex-

[2]Downloaded from http://vlsi.ece.wisc.edu/Inductwise.htm

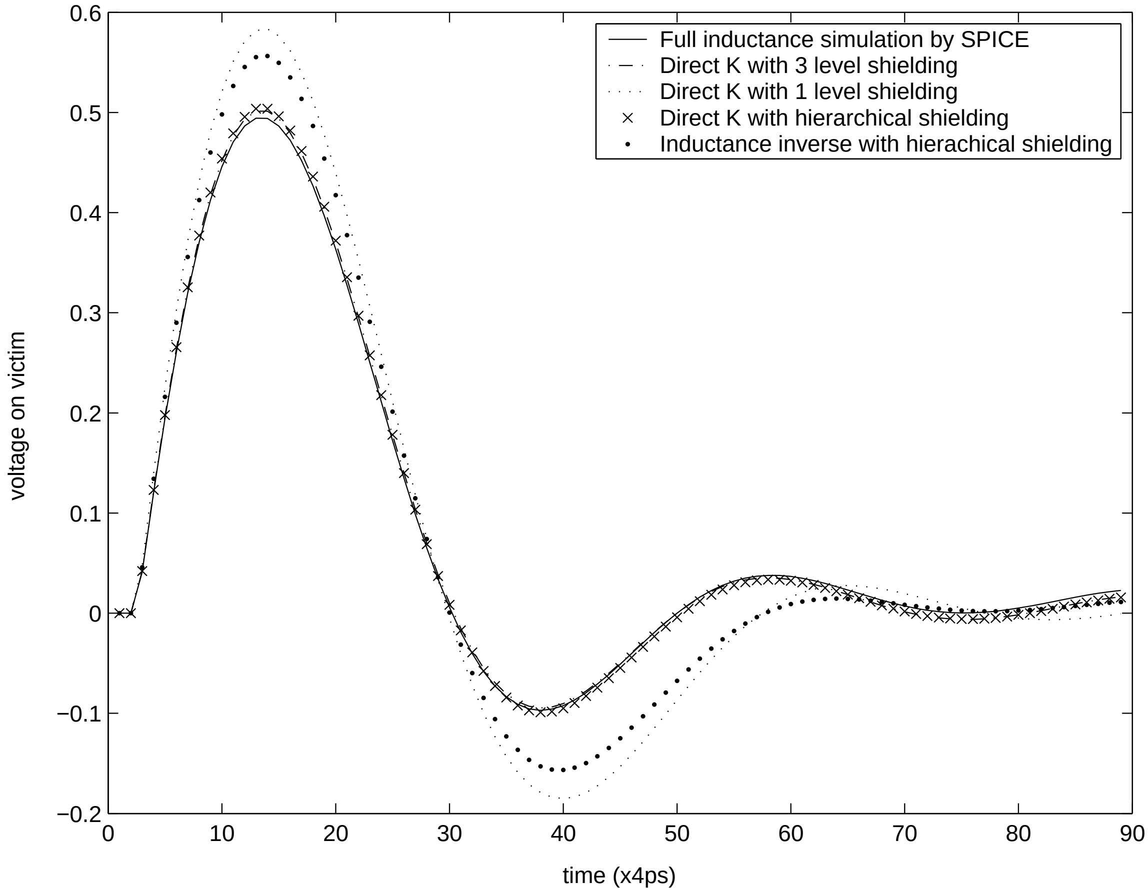

Figure 9: The waveforms on the victim got by different extraction methods

tracting the full inductance matrix. Both hierarchical shielding algorithm and 1 level simple shielding can achieve several thousand times speed up in this 300-port circuit example, while 3 level simple shielding can only achieves one hundred times speed up. However, Fig. 9 and Tab. 1 shows that use 1 level simple shielding in direct reluctance extraction may result in big error but direct extraction using hierarchical shielding algorithm is very accurate as 3 level simple shielding.

As in Tab. 1, inductance matrices inverting costs a little more extraction time than direct K extracting because of the matrices inverting time after inductance extraction. With 1 level windowing or 3 level windowing, inductance matrices inverting and direct reluctance extracting can achieve approximately the same result and same partial reluctance matrix sparsity. But inductance matrices inverting with hierarchical shielding brings much bigger error than direct K extracting on the victim conductor's voltage wave from.

6 Conclusion

In this paper, we developed a direct extraction method for partial reluctance K to capture on-chip inductance effect. The partial reluctance matrix K is calculated together with resistance R. The result of direct extraction method matches the result of inverting the partial inductance matrix. And directly extracting partial reluctance shows the physical meaning of partial reluctance in a clearer way.

One advantage of directly extracting partial reluctance is that the inductance matrix inverting can be avoided before using simulation tools based on RKC model. And the hierarchical shielding approach can speed up the partial reluctance extraction substantially without losing accuracy, which is another advantage of directly extracting partial reluctance. Since only partial reluctance has locality but partial inductance does not, hierarchical shielding approach can only be effectively used in direct K extraction.

References

[1] A.Devgan, H. Ji, and W. Dai. How to efficiently capture on-chip inductance effect: Introducing a new circuit element K. *Proc. IEEE International Conference on Computer Aided Design*, pages 150–155, Nov 2000.

[2] M. W. Beattie and L. Pileggi. Modeling magnetic coupling for on-chip interconnect. *Proc. Design Automation Conference.*, pages 335–340, June 2001.

[3] T.H. Chen and C.C.P. Chen. *User's guide for INDUCTWISE (Ver. 1.00).* University of Wisconsin-Madison, Madison, WI, 2002.

[4] T.H. Chen, C. Luk, H. Kim, and C.C.P. Chen. INDUCTWISE: inductance-wise interconnect simulator and extractor. *Proc. IEEE International Conference on Computer Aided Design.*, pages 215–220, Nov. 2002.

[5] H. A. Haus and J. R. Melcher. *Electromagnetic fields and energy.* Englewood Cliffs, NJ: Prentice-Hall, 1989.

[6] Cletus Hoer and Carl Love. Exact inductance equations for rectangular conductors with applications to more complicated geometries. *Journal of Research of the National Bureau of Standards - C. Engineering and Instrumentation.*, 69C(2):127–137, April-June 1965.

[7] H. Ji, A. Devgan, and W. Dai. KSim: A stable and efficient *RKC* simulator for capturing on-chip inductance effect. *Proc. Asia and South Pacific Design Automation Conference*, pages 379–384, Jan 2001.

[8] Gu Jiang-Chun, Wang Ze-Yi, and Hong Xian-Long. Fast determination of shielded conductors in parasitic interconnect capacitor. *Journal of Computer Aided Design and Computer Graphics.*, 12(10):721–725, Oct. 2000.

[9] M. Kamon, M.J. Tsuk, and J.K. White. FASTHENRY: A multipole-accelerated 3-D inductance extraction program. *IEEE Trans. on MTT*, pages 216–220, Sep. 1994.

[10] B. Krauter and L. Pileggi. Generating sparse partial inductance matrixes with guaranteed stability. *Proc. IEEE International Conference on Computer Aided Design.*, pages 45–52, Nov. 1995.

[11] E. B. Rosa. The self and mutual inductance of linear conductors. *Bulletin of the National Bureau of Standards.*, pages 301–344, 1908.

[12] A. E. Ruehli. Inductance calculations in a complex integrated circuit environment. *IBM Journal of Research and Development.*, pages 470–481, September 1972.

[13] K. L. Shepard and Z. Tian. Return-limited inductances: a practical approach to on-chip inductance extraction. *Proc. IEEE Custom Integrated Circuits Conference.*, pages 453–456, 1999.

[14] H. Zheng and L. Pileggi. Robust and Passive Model Order Reduction for Circuits Containing Susceptance Elements. *Proc. IEEE International Conference on Computer Aided Design.*, pages 761–766, Nov. 2002.