Solar rotation

P. Garaud* and T. Rogers†

*Department of Applied Mathematics and Statistics, University of California Santa Cruz
†HAO, NCAR, Boulder, Colorado; NSF Astronomy & Astrophysics Postdoctoral Fellow

Abstract. This paper reviews current understanding of the internal rotation of the Sun. We summarise the outcome of the latest helioseismic measurements of the angular velocity profile, and review existing dynamical models of rotation in the convection zone and in the radiative zone beneath. Finally, we discuss preliminary results along a new line of investigation.

Keywords: MHD; Solar rotation; Solar interior
PACS: 52.30.Cv; 96.60.Jw

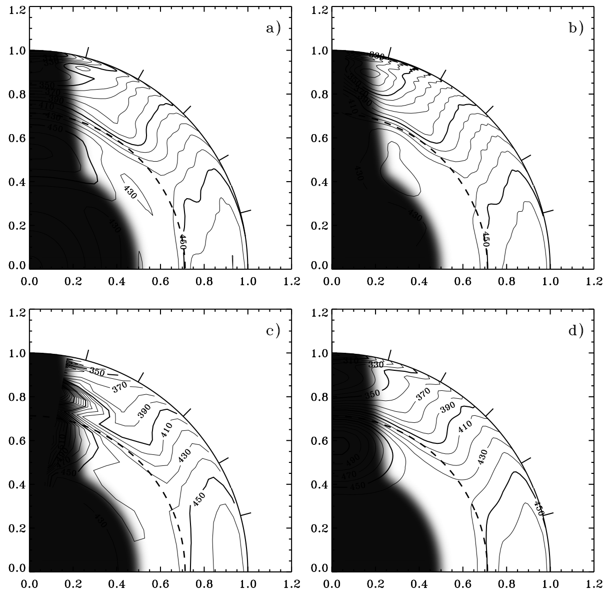
INTRODUCTION

Measurements of the surface solar differential rotation by sunspot tracking were first performed[1] by Carrington (1860). Sunspots are in fact seen to rotate with an angular velocity which is typically a few per cent faster than that of true photospheric features (Snodgrass, 1984), a result then attributed to the fact that they are anchored in more rapidly rotating sub-photospheric layers. The development of helioseismic techniques confirmed the existence of a near-surface shear layer, and revealed many other puzzling dynamical features of the solar rotation profile.

The exquisite quality of recent helioseismic observations has turned them into a laboratory for the study of fluid dynamics at asymptotically large or small values of most characteristic numbers (eg. Reynolds number, Rayleigh number, Prandtl number). Today, much of the theoretical focus is on understanding the rotation profile of the Sun: models are beginning to reach a point where quantitative comparisons with observations can be made.

In this paper, we review recent progress on measuring and modelling the solar rotation. Since detailed technical analyses of the subject have recently been published elsewhere (cf. "The Solar Tachocline" edited by Hughes, Rosner & Weiss, 2007) we select to address non-specialists by presenting a review of the key concepts and referring the reader to the adequate source where appropriate. We then propose a new line of investigation concerning the dynamics of the radiative–convective interface, and present preliminary results.

[1] Carrington noted that spots underwent increasing retrograde "longitudinal drift" with latitude when viewed from a frame rotating with a period of 25 days. He also noted the now well-known systematic poleward flows in each hemisphere.



Source: MDI/SOI website

FIGURE 1. Internal solar angular velocity $\Omega/2\pi$ (contour labels in nHz) inferred by four different inversion methods. The dashed circle indicates the base of the convection zone, and the tick marks at the edge of the outer circle are at latitudes 15°, 30°, 45°, 60°, 75°. The dark area indicates the region in the Sun where no reliable inference can be made. Adapted from Schou *et al.* (1998), see original paper for details.

HELIOSEISMIC OBSERVATIONS OF SOLAR ROTATION

Helioseismic inference of the internal rotation rate of the Sun relies on measuring the frequency difference between positively Doppler-shifted prograde sound waves and their negatively Doppler-shifted retrograde counterparts. Robust techniques for inverting these *frequency splittings* into a plausible angular velocity profile $\Omega(r,\theta)$ were developed by Gough (1985) and applied to real solar data for the first time by Duvall *et al.* (1984). Extensive reviews of helioseismic techniques and results are available elsewhere (Gough & Thompson, 1991; Christensen-Dalsgaard, 2002; Thompson *et al.* 2003).

The long-term averaged and north-south symmetric solar rotation profile is shown in Fig. 1. It is typically fitted to the simple smooth function

$$\Omega(r,\theta) = \Omega_{\rm eq}(r)(1-a_2(r)\mu^2 - a_4(r)\mu^4)\ , \text{ where } \mu = \cos\theta\ . \tag{1}$$

Very close to the surface (at $r = 0.995 r_\odot$, where $r_\odot$ is the solar radius), $\Omega_{\rm eq} \simeq 2.86\times 10^{-6}{\rm s}^{-1}$, $a_2 \simeq 0.12$ and $a_4 \simeq 0.17$ (Schou *et al.* 1998). The angular velocity increases with depth below the surface by a few percent down to about $0.9r_\odot$, in what is now commonly referred to as the near-surface shear layer. The angular velocity profile from there down to the base of the convection zone (located at $r_{\rm cz} = 0.713 r_\odot$) is roughly constant on lines inclined by 25° from the polar axis (Gilman & Howe 2003). Just above the radiative–convective interface (at $r = 0.75 r_\odot$) the rotation profile is consistent with $\Omega_{\rm eq} \simeq 2.90\times 10^{-6}{\rm s}^{-1}$, $a_2 \simeq 0.17$ and $a_4 \simeq 0.08$.

Further down, the radiative interior appears to be in a state of uniform rotation: $\Omega(r,\theta) \simeq \Omega_{\rm rz} \simeq 2.70 \times 10^{-6}{\rm s}^{-1}$ at all latitudes and radii below about $0.67 r_\odot$. A thin shear layer, the solar tachocline (Spiegel & Zahn, 1992), separates these two dynamically distinct regions. It is generally agreed that the width of the tachocline Δ is no greater than 4% of the solar radius near the equator. The detection of a possible latitudinal variation of Δ remains controversial (Charbonneau *et al.* 1999, Basu & Antia, 2003). The shear disappears in mid-latitudes (around about 30°) where the rotation of the radiative interior matches that of the convection zone.

A detailed review of the observed properties of the convection zone and the solar tachocline outlined above is given by Christensen-Dalsgaard & Thompson (2007), including recent detections of quasi-periodic spatio-temporal variations in the solar rotation profile[2].

It is probably fair to say that not a single of the observations reported here has been explained in any quantitative sense. The nature of the near-surface shear layer, its width and amplitude, have not yet been seriously addressed. Numerical simulations are shedding light on the processes involved in the equatorial acceleration of the convection zone (see below), but there again the amplitude of the observed latitudinal shear remains poorly constrained. The origin and structure of the solar tachocline is perhaps the greatest puzzle of all, and while models abound few are genuinely self-consistent and have graduated from qualitative to quantitative in terms of reproducing at the same time the observed thickness of the tachocline and the value of the internal rotation rate $\Omega_{\rm rz}$ of the bulk of the radiative zone.

MODELS OF ROTATION IN THE SOLAR CONVECTION ZONE

The outer 30% in radius of the solar interior is in a state of turbulent convection. The convective turnover timescale near the surface is only a few minutes (or in other words, four orders of magnitude faster than the solar rotation) so that the granulation pattern is more-or-less unaffected by rotational effects. The two timescales become comparable deeper into the convection zone; convective giant cells are expected to be rotationally constrained and turbulent transport becomes markedly anisotropic. This effect is thought to be the main cause for the observed equatorial acceleration of the solar convection zone, although the complete picture is naturally much more complex. For extensive reviews on the dynamics of the solar convection zone see Thompson *et al.* (2003) and Miesch (2005).

Any quasi-steady closed fluid dynamical system can in principle be understood in terms of three equilibria: thermal energy balance, angular-momentum balance and finally momentum balance in the meridional direction. The first constrains the entropy profile from the transport balance between turbulent motions and large-scale meridional flows. The second equivalently constraints the angular velocity profile. The last closes the system by relating the effect of large-scale body forces (the buoyancy force, which is indirectly related to the entropy perturbations and the Coriolis force, which is related

---

[2] These are not addressed in this paper but are nonetheless equally important and interesting.

to the angular velocity) to the divergence of the stress tensor. In many known systems, a subset of these constraints typically dominates the dynamics; whether this is the case in the solar convection zone can only be determined through numerical simulations.

Glatzmaier (1984) was the first to develop three-dimensional numerical models of turbulent anelastic convection in a spherical shell, his work ultimately leading to the development of the now widely used ASH code[3] by Miesch *et al.* (2000). Early low-resolution simulations achieved a state of "laminar" convection in which the dynamics were essentially dominated by the Taylor-Proudman constraint[4], namely $\mathbf{\Omega} \cdot \nabla \mathbf{u} \simeq 0$ where $\mathbf{u}$ is the flow velocity. The typical angular velocity profiles predicted were characteristically invariant along cylinders aligned with the rotation axis, much unlike the observed solar rotation profile. Throughout the years, increasingly high resolution simulations have been permitted by the introduction of parallel computing on large numbers of processors. Comparison between high and low resolution ("turbulent" and "laminar") simulations show a notable weakening of the Taylor-Proudman constraint and produced angular velocity profiles closer to the observed ones (Miesch *et al.* 2000). However, as argued by Brummell (2007), the turbulent cascade in the Sun extends well beyond wavenumbers for which the eddy turnover time is comparable with the average rotation rate, which is not the case in the ASH code simulations. What consequences this may have on the model predictions is not well-understood.

To answer the question raised earlier, the latest results from the ASH code (see Miesch, 2005; Miesch *et al.* 2006) suggest that each of the three equilibria listed previously could play an equally important role in determining the quasi-steady entropy and angular momentum profile of the numerical simulations. This conclusion, *if applicable to the true convection zone*, would suggest that transport by large-scale meridional flows is as important as transport by turbulent motions. This has potentially far-reaching consequences. It would imply firstly that the timescale for relaxation towards a quasi-steady-state equilibrium is the meridional flow turnover time, which is of the order of a decade or more; it is not yet numerically feasible to simulate the solar convection zone for such a long time. Secondly, it also implies that the dynamics of the solar convection are intrinsically non-local and sensitively dependent on the physics near the domain "boundaries": near the photosphere, on the dynamics of the granulation and near the radiative–convective interface, on the dynamics of the tachocline. Neither of those are currently well-modelled in the ASH code. Ongoing research is now focusing on improving the boundary conditions to mimic the presence of a strongly stratified tachocline (through imposed entropy fluctuations at the lower boundary for instance, see Miesch *et al.* 2006) and of the granulation (as reported by Miesch in this meeting, through stochastic noise near the outer boundary), with some success.

A different approach to modelling the solar convection zone has been pursued in parallel for about as long as the numerical efforts described above. Building on the successful formulation of mixing-length theory (Böhm-Vitense, 1958) which provides a plausible

---

[3] Anelastic Spherical Harmonics

[4] This constraint is related to the conservation of vorticity in the limit where the dominant momentum balance is between pressure and Coriolis forces (i.e. when turbulent stresses or entropy fluctuations are negligible)

parametrised prescription of the heat flux transported by small-scale turbulent motions, many attempts have been made at modelling the equivalent transport of momentum by turbulent Reynolds stresses[5].

Closure models attempt to construct plausible prescriptions for the Reynolds stress tensor **R** from heuristic arguments (first-order closure), or by solving a parametrised evolution equation for **R** in parallel with the mean flow equations (second-order closure). Early attempts constructed a Reynolds stress tensor from correlations of linearly unstable eigenmodes (Gough, 1978; Hathaway, 1984), with amplitudes selected to match predictions of mixing-length theory. Other methods construct the Reynolds stress tensor from dimensional analysis and geometrical arguments (Durney & Spruit, 1979; Kitchatinov & Ruediger, 1993).

Recently, Rempel (2005) carried out a systematic study of the solar rotation profile using a first-order closure model. He was able to test not only the effect of the selection of the unknown parameters on the predicted profile, but also that of the selection of the background state. Notably, his work provided the first clear indication of the extent of the influence of the tachocline on the rotation profile in the convection zone, highlighting its role as an active boundary layer and to some extent, the futility of trying to model the convection zone *in detail* without correctly dealing with the stably stratified region below. Generally speaking, however, it must be noted that many Reynolds stress closure models applied in the astrophysical context have *not* been tested against systems for which there exists either numerical or experimental data. Thus their absolute predictive power is questionable. A second-order closure model of turbulent stresses has recently been proposed by Ogilvie (2003) and was successfully tested against experimental data of Couette-Taylor flows by Garaud & Ogilvie (2005). Preliminary analysis of the performance of the model in the context of Rayleigh-Benard convection yields promising results (Miller & Garaud, this volume).

MODELS OF ROTATION IN THE SOLAR RADIATIVE ZONE AND THE TACHOCLINE

The transition from a nearly adiabatic to a strongly stratified background entropy gradient[6] across the radiative–convective interface causes a dramatic change in the nature of fluid motions: radial velocities in the radiative zone are rapidly damped so that turbulent or wave-like flows are essentially horizontal, while the only large-scale flows allowed to penetrate the interior have global turnover timescales longer than the thermal diffusion time (eg. $\sim 10^4$ years across the depth of the tachocline).

The first model of the tachocline (Spiegel & Zahn 1992) originally proposed that

---

[5] The components of the Reynolds stress tensor **R** are the quadratic correlations $R_{ij} = \overline{u'_i u'_j}$ where the overbar denote a statistical, spatial or temporal mean with suitable mathematical properties. The divergence of the stress tensor represents the mean effect of the small-scale motions on the momentum equation.

[6] The ratio N^2/Ω^2, where N is the Brunt-Väisälä frequency, varies from a value of about -0.02 at $r = 0.715 r_\odot$ to 5×10^4 at $r = 0.71 r_\odot$ (cf. Model S of Christensen-Dalsgaard, Gough & Thompson, 1991).

the two-dimensional nature of strongly stratified turbulence was in fact the cause of the sharpness of the transition from the sustained latitudinal shear observed in the convection zone to the near-uniform rotation of the radiative zone. By parametrising the effects of the turbulent motion with an anisotropic viscosity[7] they showed that the rotation profile of the radiative zone could be qualitatively reproduced provided turbulent viscosity in the latitudinal direction was orders of magnitude larger than in the vertical direction. This idea is possibly the simplest and most pleasing that has been proposed to date, and has the added advantage of permitting strict quantitative predictions that can be directly compared with observations. However, it has been criticized for its simplified treatment of turbulent transport.

McIntyre (2003) reports that turbulent transport in a stratified fluid as measured in the Earth's stratosphere can only be reconciled with the conservation of potential vorticity[8] whereas the parametrisation proposed by Spiegel & Zahn assumes the conservation of angular momentum. In addition, Tobias, Diamond & Hughes (2007) showed that if the turbulence is not strictly hydrodynamic (which is most likely the case of the Sun) then tiny seed magnetic fields are amplified until turbulent Lorentz stresses exactly cancel out turbulent Reynolds stresses, to the extent that the resulting transport properties reduce to the original microscopic viscous momentum flux. These two papers illustrate the difficulty in constructing a model that adequately predicts turbulent stresses in the tachocline. Note that other hydrodynamic models of the tachocline have also been proposed (notably involving the action of gravity waves) although none have yet reached a stage where strict quantitative comparisons with helioseismic observations have been made. The reader is referred to the review of Zahn (2007) for details.

Moving away from models which involve angular-momentum transport by Reynolds stresses Gough & McIntyre (1998) argued that the presence of a large-scale field within the solar radiative zone is the only possible explanation for the observed uniform rotation. Indeed, in the absence of diffusion, strong meridional flows or strong turbulent motions, the longitudinal component of the magnetic induction equation suggests that the field and the fluid must relax to a quasi-steady state of *iso*rotation which requires that Ω should be constant along field lines (Ferraro, 1937). If the field is entirely confined within the radiative zone then uniform rotation naturally ensues, but if field lines overlap with the differentially rotating convection zone then the interior must also rotate differentially: confining the field below the convection zone appears to be an essential part of this class of models (McGregor & Charbonneau, 1999; see the review by Garaud, 2007a for more details).

Gough & McIntyre proposed that nonlinear interactions with *large-scale* flows driven by turbulent stresses in the convection zone and burrowing into the radiative zone could confine the field strictly below the base of the tachocline. Brun & Zahn (2006) and Garaud (2007b) studied this idea numerically, the former using the time-dependent ASH code and the latter with a steady-state code; both investigated the dynamics of the

---

[7] The form of viscosity adopted is such that the momentum flux is the product of the angular-velocity gradient and a diagonal matrix whose vertical element differs, and is very much less than, the two equal horizontal components.

[8] The component of vorticity along the gradient of entropy.

radiative zone only, and the physics of the radiative–convective interface are modelled through boundary conditions. Brun & Zahn considered a model in which the interface is represented as an impermeable and no-slip boundary rotating differentially with the angular velocity profile of the convection zone. Starting with an initially confined dipolar magnetic field, they found that the field lines inevitably always diffuse through the interface and that the rotation profile of the interior evolves toward a differentially rotating Ferraro state. They conclude that field confinement by the Gough & McIntyre mechanism is not possible.

Garaud (2007b) recovers the same result under the same set of boundary conditions. However, she interprets the conclusion as a direct consequence of the artifical impermeability condition, which constrains the meridional flow velocities to scale as Ekman-Hartman flows and limits their amplitude to very low values with an associated magnetic Reynolds number well below unity. Since the Gough & McIntyre model explicitly relies on flows downwelling from the convective zone to confine the field, Garaud (2007b) also tested another set of boundary conditions for which an imposed single-cell equatorward meridional flow is pumped through the outer boundary. In this case, the flow velocities remain high and field confinement appears to be possible but a different problem occurs (see her Fig. 2): transport of negative angular momentum within the radiative zone from high to low latitudes by the single-cell flow is so efficient that the predicted interior velocity drops well-below the observed value. Thus the selection of the meridional flow profile at the radiative–convective interface is crucial to the angular-momentum balance of the whole interior.

There are two important conclusions to draw from the outcome of these simple numerical simulations. Firstly, the dynamics of the radiative interior are sensitively dependent on the assumed interfacial conditions, even on a mere qualitative level. One may at this point reflect on the futility of trying to model the radiative interior and the tachocline without correctly dealing with the convective region above. Secondly, it illustrates the importance of simple specific quantitative diagnostics (such as for example the value of the bulk rotation rate in the radiative zone) in addressing the quality of a model: many existing theories of the tachocline, which satisfy themselves with reproducing its broad qualitative properties, are seen to fail as soon as quantitative comparisons with observations are made.

Finally, the question of whether the model proposed by Gough & McIntyre can indeed explain the observed rotation profile in the radiative zone remains open. While it may indeed hold the key to the field confinement problem, only detailed quantitative analyses will reveal whether their model is the whole story, or only part of the story. A more conclusive answer will (hopefully) be provided in a forthcoming paper.

A DIFFERENT LOOK AT THE TACHOCLINE

While models of the tachocline involving large-scale flows and large-scale fields are still under investigation, we have also begun to revisit the problem from an alternative point of view.

We first address the following question: in the *absence* of large-scale flows, what is the fate of a large-scale poloidal field initially confined within the radiative zone? According

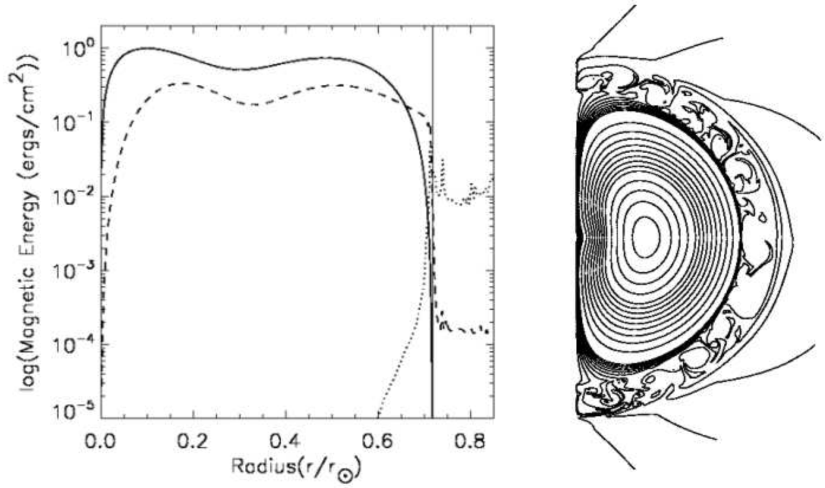


FIGURE 2. Left-hand side: Normalised magnetic energy integrated over a shell as a function of radius at time $t = 0$ (solid line) and at a later time (dashed and dotted lines) corresponding to about one diffusion time across the tachocline. The dashed line shows the energy contains in the largest-scale latitudinal mode only while the dotted line shows the energy contained in all other modes. The vertical line marks the edge of the convection zone. Right-hand side: a few selected magnetic field lines at the later time, showing clearly the apparently confined large-scale field in the radiative interior, and the small-scale nature of the field in the convection zone.

to the Cartesian-box simulations of Tobias *et al.* (2001), one should still expect the bulk of the field to remain confined below the convection zone. The interaction between strong turbulent motions and magnetic fields leads to flux expulsion, whereby fields on scales larger than the typical eddy-scale are shuffled out of the turbulent region. While their simulations represent a small three-dimensional region along the radiative–convective interface, we perform two-dimensional numerical simulations of the entire interior, starting with a large-scale dipolar field initially entirely embedded within the radiative zone[9]. The numerical algorithm has been modified from that of Rogers & Glatzmaier (2005) to include magnetic fields. The system studied is non-rotating, which guarantees the absence of any large-scale flows; dynamo action is also forbidden. The strength of the magnetic field was chosen to be only a few tens of Gauss in the region of the tachocline, so that the convective motions are only very weakly influenced by it. The results are shown in Fig. 2. The right-hand figure reveals the bimodal nature of the internal field, with a large-scale and very slowly evolving dipolar component below the overshoot layer, while any field line diffusing into the convection zone is very rapidly

---

[9] Our system is geometrically different from the one studied by Tobias *et al.* (2001). The dipolar nature of the field garantees that the *total* magnetic flux through the radiative–convective interface (and any a concentric surface) is identically zero, which is equivalent to their simulations. The difference lies in the fact that in the absence of convective motions field lines would relax to a state with a non-zero radial field nearly everywhere along the interface. Moreover, every single field line near the interface in one hemisphere connects to its opposite-hemisphere “twin” somewhere in the deep and stably stratified interior. Magnetic tension and buoyancy effects are very likely to play a role in the global field evolution (Weiss, 2006).

advected, stretched and forced to reconnect with other field lines by the turbulent eddies. The left-hand figure shows the magnetic energy in the lowest spatial mode (the $l = 1$ mode, varying as $\cos\theta$, as a dashed line), as well as that in higher-order modes ($l = 2$ to $l = 800$, as a dotted line). It illustrates both the partial confinement of the large-scale field below the overshoot layer (note the reduction by three orders of magnitudes of the amplitude of the $l = 1$ mode across the overshoot layer) and the transfer of most of the magnetic energy to small-scales within the turbulent region.

The picture which emerges from these simulations is quite different from the one implied by the numerical solutions reported in the previous section. There, field lines which connect to the outer boundary of the numerical domain (the radiative–convective interface) are forced to rotate with the angular velocity of the convection zone at that given latitude, hence the inevitable propagation of the shear into the radiative zone. Here, our simulations show instead that field lines which enter the convection zone are far from static, and undergo vast horizontal excursions before reconnecting with other field lines in the overshoot layer. As a result, the large-scale field in the interior supports the propagation of *stochastically* excited Alfvén waves, with associated angular-momentum transport properties which are undoubtedly very different (eg. phase-mixing is likely to be much more important).

Testing the consequence of these effects on the rotation profile of the solar interior is extremely difficult. The only correct way of doing it would be to study the full three-dimensional problem including both the radiative zone and the convection zone, for a long-enough time to permit a quasi-steady equilibrium to be reached. This is not numerically feasible in the foreseeable future.

Instead, we perform a preliminary investigation of the consequence of *assuming* field confinement by turbulent overshoot on the dynamics of the radiative interior, focusing on quantitative predictions for the angular velocity profile. The numerical model used towards this goal is the same as the one used by Garaud (2007b) for which some results were described in the previous section. However, in this case the boundary conditions on the magnetic field are modified to $B_r = 0$ and $B_\phi = 0$, where B_r and B_ϕ are the radial and azimuthal components of the magnetic field respectively; this artificial set of boundary conditions supposedly mimics the destruction of the field at the radiative–convective interface. For the purpose of the following discussion it is also important to note that the equations solved use artificially enhanced diffusivities, as it is common in numerical simulations. More precisely, the viscosity, magnetic diffusivity and thermal diffusivity are those appropriate to a standard solar model, but uniformly multiplied by the same constant factor f. We then seek to understand the asymptotic behaviour of the numerical solutions as $f \to 1$.

A typical outcome of the numerical simulations is shown in Fig. 3. The predicted angular velocity profile naturally reproduces the existence of a tachocline for low enough values of the magnetic diffusivity, as found by Ruediger & Kitchatinov[10] (1997) using

[10] Ruediger & Kitchatinov (1997) studied the effect of a fixed confined dipolar field on the differential rotation. In their work, only the azimuthal components of the field and the flow are calculated, thus neglecting meridional flows, their role in transporting angular momentum and their nonlinear interaction with the field.

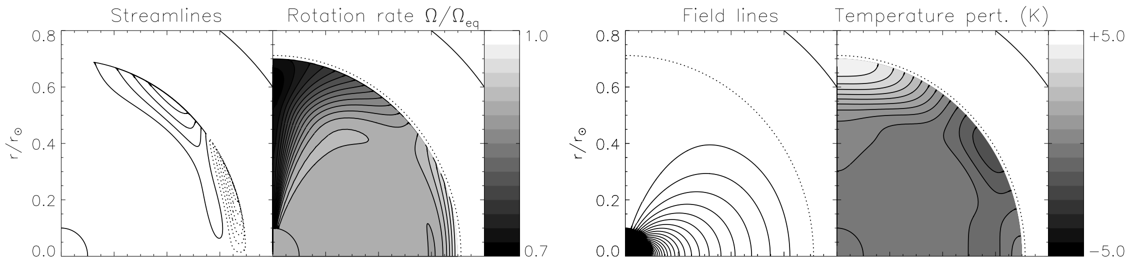


FIGURE 3. Numerical solution for the dynamics of the radiative zone with $f = 5\times10^7$. The outer edge of the computational domain is at $r = 0.7r_\odot$, just slightly under the assumed position of the overshoot layer. For this simulation, the boundary conditions applied at $0.7r_\odot$ are: $u_r = u_\theta = 0$ and $u_\phi = u_{cz}$ where u_{cz} is the azimuthal velocity profile observed in the convection zone. In the first panel, solid lines represent counter-clockwise flow and dotted lines represent clockwise flow.

a reduced model with similar boundary conditions; a ubiquitous characteristic feature of this type of model is the existence of slowly rotating polar regions throughout the interior. Meridional flows are seen to be confined by the magnetic field to the region of the tachocline instead of burrowing deeply into the radiative zone; this property was predicted by Gough & McIntyre, and is required by observational limits on mixing of chemical elements between the radiative and convective regions of the Sun.

The predicted value of the angular velocity profile in the interior Ω_{in} can be used, as mentioned earlier, as a simple diagnostic of angular momentum transport in the model[11]. Since the *large-scale* dynamics of the system can be calculated exactly, quantitative comparisons with observations can be made, and it had been our hope to identify discrepancies with the observed profile as evidence for missing small-scale transport in the model. As it turns out, the concept is indeed useful, but the conclusions we now draw differ from those presented in Cambridge at the time of the meeting.

Fig. 4 shows the model predictions for $\Omega_{in}/\Omega_{eq}(r_{cz})$ as a function of the factor f, for three different values of the internal field strength. For the largest values of f, the magnetic field merely diffuses through the fluid without affecting it. As f decreases, the nonlinear interaction between the field and the flows begins to dominate the interior dynamics. Since Ω_{in} is determined by the balance between advection by large-scale meridional flows and Lorentz stresses we also show the dependence of Ω_{in} on the assumed dipolar field strength.

Preliminary results shown in Cambridge (eg. the open symbols on Fig. 4) were consistent with $\Omega_{in}(f)$ curves flattening out as $f \to 1$, suggesting that the system may be approaching an asymptotic solution. The asymptotic value of Ω_{in} was argued to be consistent with observations for a range of plausible values of the internal field, and conclusions were drawn therefrom. However, more recent simulations at higher resolution and even lower values of f show that the system enters a new regime instead in which a super-rotating region appears near the inner core (see Fig. 4). We attribute

[11] The observed interior angular velocity Ω_{rz} is roughly equal to 93% of the equatorial angular velocity at the base of the convection zone $\Omega_{eq}(r_{cz})$. For comparison, Spiegel & Zahn predict that $\Omega_{in} = 0.91\Omega_{eq}(r_{cz})$

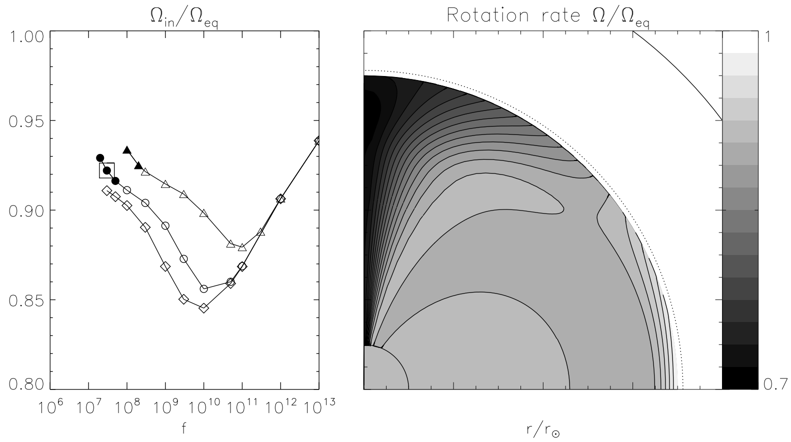


FIGURE 4. On the left, we show the ratio of the angular velocity $\Omega_{\rm in}$ measured on the inner boundary at the equator to that of the outer boundary at the equator for three different interior field strengths: $B_0 \simeq 20$G (diamonds), $B_0 \simeq 60$G (circles) and $B_0 \simeq 600$G (triangles) where B_0 is the amplitude of the magnetic field at the pole at $r = 0.7r_\odot$ that the solutions would have were they purely diffusive. The open symbols are similar to those shown at the Cambridge meeting for a slightly different background state, and the filled symbols are higher resolution simulations that were performed since. The open square marks the parameters corresponding to the solution shown in the right-hand figure, which clearly illustrates the emergence of a slightly super-rotating layer.

the emergence of this super-rotation to meridional flows burrowing from the outer equatorial regions towards the interior and forced to deposit their angular momentum before returning. These flows are preferentially aligned with the field lines and form magneto-viscous internal layers scaling as $(\nu\eta)^{1/4}$. This kind of dynamical behaviour was already observed by Dormy, Cardin & Jault (1998) and by Garaud (2002) in the context of an incompressible system, and it is in hindsight not entirely surprising to find it here also.

Further analysis will be required to identify the role of these internal layers in this kind of model. Whether they are likely to exist in the Sun is a different question: their formation and maintenance relies on a fairly tenuous magneto-viscous balance of forces. However, phase-mixing between field lines is likely to supersede the kind of dynamics that these simple numerical solutions imply. This is already illustrated in Fig. 2, where some of the effects of phase-mixing on the magnetic field itself can just be discerned near the radiative core. The role of phase-mixing on solar roation is worth revisiting.

ACKNOWLEDGMENTS

P. G. acknowledges funding from NSF-AST-0607495. T.R. is supported by an NSF Astronomy and Astrophysics Postdoctoral Fellowship under award 0602023 and by NASA-NNG06GD44G. We thank N. Brummell, G. Glatzmaier, D. Gough, M. McIntyre and S. Tobias for interesting discussions, and J.-D. Garaud for his contribution to code development. P.G.'s results were obtained with the Pleiades cluster purchased with NSF-MRI-0521566.

REFERENCES

S. Basu and H. M. Antia, *Astrophys. J.*, **585**, 553–565 (2003).
E. Böhm-Vitense, *Zeit. für Astrophys.*, **46**, 108–143 (1958).
N. H. Brummell, in *Proceedings of IAU Symposium 239*, edited by F. Kupka, I. Roxburgh and K. Chan, Cambridge University Press, Cambridge, 2007, pp. 417–424
A. S. Brun and J.-P. Zahn, *Astron. Astrophys.*, **467**, 665–674 (2006).
R. C. Carrington, *Mon. Not. Roy. Astron. Soc.*, **20**, 254–257 (1860).
P. Charbonneau, *et al.*, *Astrophys. J.*, **527**, 445–460 (1999).
J. Christensen-Dalsgaard, D. O. Gough and M. J. Thompson, *Astrophys. J.*, **378**, 413–437 (1991).
J. Christensen-Dalsgaard, *Rev. Mod. Phys.*, **74**,1073–1129 (2002).
J. Christensen-Dalsgaard and M. J. Thompson in *The Solar Tachocline*, edited by D. W. Hughes, R. Rosner and N. O. Weiss, CUP, Cambridge, 2007, pp. 53–85.
E. Dormy, P. Cardin and D. Jault, *Earth & Planetary Sci. Let.*, **160**, 15–30 (1998)
B. R. Durney and H. C. Spruit, *Astrophys. J.*, **234**, 1067–1078 (1979).
T. L. Duvall Jr *et al.*, *Nature*, **310**, 22–25 (1984)
V. C. A. Ferraro, *Mon. Not. Roy. Astron. Soc.*, **97**, 458–472 (1937).
P. Garaud, *Mon. Not. Roy. Astron. Soc.*, **329**, 1–17 (2002).
P. Garaud and G. I. Ogilvie, *J. Fluid Mech.*, **530**, 145–176 (2005).
P. Garaud, in *The Solar Tachocline*, edited by D. W. Hughes, R. Rosner and N. O. Weiss, CUP, Cambridge, 2007a, pp. 147–181
P. Garaud, in *Proceedings of the 5th Potsdam Thinkshop on Meridional Flows, Differential Rotation and Stellar Activity*, to appear in *Astron. Nach.* (2007b).
P. A. Gilman and R. Howe, in *Local and Global Helioseismology: The Present and the Future*, edited by H. Sawaya-Lacoste, ESA SP-517, Noordwijk, Netherlands, 2003, pp. 283–285.
G. A. Glatzmaier, *J. Comp. Phys.*, **55**, 461–484 (1984).
D. G. Gough, in *Proc. EPS Workshop on Solar Rotation* edited by G. Belvedere & L. Paternò, Catania University Press, 255–268 (1978)
D. O. Gough, *Solar Phys.*, **100**, 65–99 (1985).
D. O. Gough and M. J. Thompson, in *Solar interior and atmosphere*, University of Arizon Press, Tucson, 1991, pp. 519–561
D. O. Gough and M. E. McIntyre, *Nature*, **394**, 755–758 (1998).
D. H. Hathaway, *Astrophys. J.*, **276**, 316–324 (1984).
L. L. Kitchatinov and G. Ruediger, *Astron. Astrophys.*, **269**, 581–588 (1993).
K. B. MacGregor and P. Charbonneau, *Astron. Astrophys.*, **519**, 911–917 (1999).
M. E. McIntyre, in *Stellar Astrophysical Fluid Dynamics*, edited by M. J. Thompson and J. Christensen-Dalsgaard, CUP, Cambridge, 2003, pp. 111–130.
M. S. Miesch, *et al.*, *Astrophys. J.*, **532**, 593–615 (2000).
M. S. Miesch, *Liv. Rev. Solar Phys.*, **2**, (2005)
M. S. Miesch, A. S. Brun and J. Toomre, *Astrophys. J.*, **641**, 618–625 (2006).
G. I. Ogilvie, *Mon. Not. Roy. Astron. Soc.*, **340**, 969–982 (2003).
E. N. Parker, *Astrophys. J.*, **408**, 707–719 (1993).
M. Rempel, *Astrophys. J.*, **622**, 1320–1322 (2005).
T. M. Rogers and G. A. Glatzmaier, *Astrophys. J.*, **620**, 432–441 (2005).
G. Ruediger and L. L. Kitchatinov, *Astron. Nach.*, **378**, 273–279 (1997).
J. Schou, *et al.*, *Astrophys. J.*, **505**, 390–417 (1998).
H. B. Snodgrass, *Solar Phys.*, **94**, 13–31 (1984).
E. A. Spiegel and J.-P. Zahn, *Astron. Astrophys.*, **265**, 106–114 (1992).
M. J. Thompson, J. Christensen-Dalsgaard, M. S. Miesch, and J. Toomre, *Ann. Rev. Astron. Astrophys.*, **41**, 599–643 (2003).
S. M. Tobias, N. H. Brummell, N. H. Clune and J. Toomre, *Astrophys. J.*, **549**, 1183–1203 (2001).
S. M. Tobias, P. H. Diamond and D. W. Hughes, *Astrophys J.*, **667**, L113–L115 (2007)
N. O. Weiss, in *Space Science Reviews*, **124**, 13–22 (2006)
J.-P. Zahn, in *The Solar Tachocline*, edited by D. W. Hughes, R. Rosner and N. O. Weiss, CUP, Cambridge, 2007, pp. 89–107