

# Constraints on angular momentum transport in the Sun from simulations of the tachocline

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I present a new numerical tool for studying the interaction of meridional flows and magnetic fields, and study their role in establishing angular-momentum balance in the solar radiative zone. Quantitative comparisons with helioseismic observations provide stringent constraints on existing models of the dynamics of the solar interior.

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## 1 Introduction

The tachocline is a thin shear layer located near the interface between the uniformly rotating radiative zone and the differentially rotating convective zone in the Sun. Indeed, helioseismic observations (Schou *et al.* 1998; Gough 2007) reveal that the base of the convection zone has an angular velocity profile  $\Omega_{cz}(\theta)$  consistent with

$$\Omega_{cz}(\theta) = \Omega_{eq}(1 - a \cos^2 \theta - b \cos^4 \theta) \text{ with}$$

$$\Omega_{eq} = 2.9 \times 10^{-6} \text{ s}^{-1}, \quad a = 0.17, \quad b = 0.08, \quad (1)$$

where  $\theta$  is the co-latitude, whereas less than 30,000 km deeper (for  $r < 0.67r_{\odot}$ ) the radiative zone is rotating with a constant angular velocity

$$\Omega_{rz}(\theta) = 0.93\Omega_{eq}. \quad (2)$$

The origin of this sharp transition has been a puzzle for nearly two decades.

The solar tachocline is thought to be a permanent feature of the Sun, operating the dynamical transition between the radiative and convective regions by slowly adjusting the rotation rate of the interior to that of the surface as it is spun down by magnetic braking (Spiegel 1972, 2007). Dynamical processes in the tachocline are also thought to reduce the gravitational settling of helium with respect to hydrogen (Elliott & Gough 1999), to enhance the circulation of light elements (e.g. Li or Be) between the surface and their nuclear-burning region, and last but not least, to play a key role in the generation of the solar magnetic cycle (Parker, 1993).

There exist no self-consistent quantitative model of the solar tachocline successfully capable of explaining helioseismic observations. At the heart of the problem is the difficulty in quantifying the turbulent hydromagnetic stresses responsible for angular-momentum transport in this dynamically complex region of the Sun.

The equation governing the conservation of the angular-momentum  $L = r \sin \theta u_{\phi} = r^2 \sin^2 \theta \Omega$  can be written as

$$\begin{aligned} \frac{\partial}{\partial t}(\rho L) + \nabla \cdot [\rho \mathbf{u} L + \rho r \sin \theta \langle u'_{\phi} \mathbf{u}' \rangle \\ - \frac{r \sin \theta}{4\pi} B_{\phi} \mathbf{B} - \frac{r \sin \theta}{4\pi} \langle B'_{\phi} \mathbf{B}' \rangle \\ - \rho r^2 \sin^2 \theta \nu \nabla \Omega] \end{aligned} \quad (3)$$

where  $\rho$  is the density,  $\mathbf{u} = (u_r, u_{\theta}, u_{\phi})$  is the flow velocity in a spherical coordinate system  $(r, \theta, \phi)$ ,  $\mathbf{B} = (B_r, B_{\theta}, B_{\phi})$  is the magnetic field and  $\nu$  is the microscopic viscosity. In this expression the flow and field are viewed as the sum of a large-scale, slowly varying mean term and of a small-scale rapidly varying perturbation (denoted by a prime). Equation (3) explicitly reveals the angular-momentum flux to have five essential contributions: advection by large-scale flows, turbulent Reynolds stresses, large-scale and small-scale Lorentz stresses and finally diffusion by microscopic viscosity. There are many possible sources for turbulent transport in the tachocline region: from the base of the convection zone downward, overshooting convective plumes, hydromagnetic instabilities and finally nonlinear and linear waves of mixed nature. Quantifying their contribution to the total angular-momentum balance is a formidable task, which has been undertaken by many using a variety of different methods such as 2-D direct numerical simulations (Rogers & Glatzmaier 2006, Rogers, Glatzmaier & Jones 2006, Tobias *et al.* 2007), 3-D numerical simulations (Brummell, Clune & Toomre 2002, Miesch 2003, Miesch, Gilman & Dikpati 2007), quasi-nonlinear stability theory (Charbonneau, Dikati & Gilman 1999, Garaud 2001a) and finally turbulence closure models (Rempel 2005, Leprovost & Kim 2006 and other articles by the same authors).

Here I approach the problem from a different angle, noting that while turbulent and wave-induced hydromagnetic stresses are difficult to quantify in parameter regimes appropriate for the tachocline, all of the other large-scale angular-momentum transporters can be realistically simulated. By

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studying the difference between the model predictions and helioseismic observations one may be able to constrain theories of the remaining unknown stresses.

## 2 Model equations

In this paper I consider the *laminar* and long-term dynamics of the radiative zone under the influence of magnetic fields and large-scale flows. For simplicity, I assume that the flow and field are axially symmetric, in a quasi-steady state, and that the perturbations to the background hydrostatic equilibrium are small enough for linearization to be appropriate. The system is then governed by the following equations:

$$\begin{aligned}
2\bar{\rho}\bar{\Omega} \times \mathbf{u} &= -\nabla\tilde{p} - \tilde{\rho}\nabla\bar{\Phi} + \mathbf{j} \times \mathbf{B} + f\nabla \cdot \Pi \\
\nabla \cdot (\bar{\rho}\mathbf{u}) &= 0 \\
\bar{\rho}\bar{T}\mathbf{u} \cdot \nabla\bar{s} &= f\nabla \cdot (\bar{k}\nabla\tilde{T}) \\
\frac{\tilde{p}}{\bar{p}} &= \frac{\tilde{\rho}}{\bar{\rho}} + \frac{\tilde{T}}{\bar{T}} \\
\nabla \times (\mathbf{u} \times \mathbf{B}) &= f\nabla \times (\bar{\eta}\nabla \times \mathbf{B}) \\
\nabla \cdot \mathbf{B} &= 0
\end{aligned} \tag{4}$$

representing respectively momentum, mass and thermal energy conservation, the equation of state, the induction equation and finally the solenoidal condition. Here,  $\bar{\Omega}$  is the background angular velocity which, when integrated over a sphere, has the same specific angular-momentum as that of the convection zone. The thermodynamical quantities  $p$ ,  $s$  and  $T$  have their usual meaning,  $\mathbf{j}$  is the current density,  $\Pi$  is the viscous stress tensor, and  $\Phi$  is the gravitational potential. Finally,  $\eta$  is the magnetic diffusivity while  $k$  is the thermal conductivity.

Quantities denoted with overbars are spherically symmetric background quantities obtained by interpolating Model S of Christensen-Dalsgaard, Gough & Thompson (1991) upon my own numerical mesh, and the diffusivities  $\nu$ ,  $\eta$  and  $k$  have been derived by Gough (2007). Note that the same constant factor  $f$  multiplies all of the diffusive terms. This artificially enhances the diffusivities to such a point where thin boundary layers can be numerically resolved while preserving solar values of the Prandtl number, magnetic Prandtl number and Roberts number. I do not view it as turbulent diffusivities, but strive instead to study the asymptotic behaviour of the solutions as  $f$  is gradually reduced to unity.

A detailed description of the numerical method of solution for this particular problem will be published in a forthcoming paper; it is closely related to that described and used in my PhD work (Garaud, 2001b; Garaud, 2002).

## 3 Model setup

Gough & McIntyre (1998) proposed the first self-consistent magnetic tachocline model. They argue that the uniform rotation of the bulk of the solar radiative zone cannot be explained without the presence of a large-scale primordial

field, which must be entirely confined within the radiative zone. Indeed, preventing magnetic field lines from overlapping with the convection zone is crucial since Alfvénic angular-momentum transport could otherwise rapidly impose a differentially rotating Ferraro state throughout the interior (McGregor & Charbonneau, 1999). To confine the field, Gough & McIntyre consider large-scale flows driven by turbulent stresses within the convection zone, which are known to burrow downwards into the radiative zone (Spiegel & Zahn 1992; Garaud & Brummell, 2007). These flows are thought to confine the primordial field below a very thin diffusion layer located near the bottom of the tachocline. Meanwhile, the field also prevents meridional flows from burrowing too deeply into the radiative zone thereby satisfying observational limits on transport of light elements.

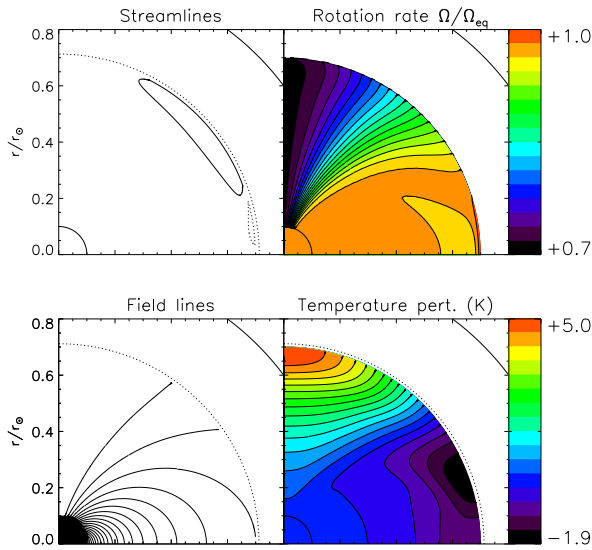
This idea can be tested using the equations and numerical method described in §2. There remains one important issue to discuss, namely the selection of boundary conditions that best represent the Gough & McIntyre model. Indeed, since Gough & McIntyre consider the Sun as a whole, they do not need to specify conditions near the radiative-convective interface. The current numerical model on the other hand explicitly excludes the convection zone and overshoot layer by considering only the dynamics of the radiative interior in the domain  $r \in [0.1, 0.7]r_{\odot}$ ; “interfacial” conditions must be applied to represent the effect of the overshoot layer on the underlying radiative zone. Since in any steady-state numerical calculation the boundary conditions are *essential* in determining the nature of the solutions, these must be selected with the utmost care.

Boundary conditions on the inner core (for  $r < 0.1r_{\odot}$ ) are chosen in such a way as to limit its artificial influence as much as possible. Accordingly, it is assumed to be perfectly thermally conducting by requiring that  $\nabla^2\tilde{T} = 0$  within and deriving matching conditions for  $\tilde{T}$  and its derivative at the boundary. It is impermeable, and assumed to be entirely stress-free. Finally, the magnetic field is assumed to diffuse easily into the excluded inner sphere ( $\nabla^2\mathbf{B} = 0$  within) and matches on to a point dipole as  $r \rightarrow 0$ .

Boundary conditions near the base of the overshoot layer are selected to mimic its effect on the radiative zone dynamics. It seems natural to set  $u_{\phi} = r \sin\theta\Omega_{cz}(\theta)$  at  $r = 0.7r_{\odot}$  where  $\Omega_{cz}(\theta)$  is given in equation (1); following Garaud & Brummell (2007), I also require that  $\bar{p}\bar{h}u_r = \kappa\nabla\tilde{T}$  at the boundary (where  $\bar{h}$  is the specific enthalpy, and  $\kappa$  is the thermal diffusivity). A variety of plausible prescriptions for  $u_r$  and  $u_{\theta}$  at  $0.7r_{\odot}$  are now considered.

## 4 Model results

Fig. 1 is a typical simulation output when the outer boundary of the domain is assumed to be impermeable and no-slip ( $u_r = u_{\theta} = 0$  at the interface), for value of the constant  $f = 10^8$  which corresponds to an Ekman number  $E_{\nu} = \nu/r_{\odot}^2\Omega_{eq} \simeq 10^{-8}$ , and a magnetic Ekman number  $E_{\eta} = \eta/r_{\odot}^2\Omega_{eq} \simeq 10^{-5}$  (both well into the asymptotically



**Fig. 1** Example of numerical solution for impermeable and no-slip conditions,  $f = 10^8$  and  $B_r(0.1r_\odot, 0) = 2.2\text{T}$ . The top-left quadrant shows selected meridional flow streamlines, with clockwise flow shown in dotted lines and counter-clockwise flow shown in solid lines. The top-right quadrant shows the angular velocity profile. The bottom-left quadrant shows the magnetic field lines, and the bottom right quadrant shows the temperature perturbations.

small regime). The strength of the magnetic field at the pole on the inner boundary is about 2.2T, which in the absence of flows would yield a field of about 70 G near the outer boundary at the same latitude. In this particular setup, the flows are induced by Ekman-Hartman pumping (Acheson & Hide, 1973), and scale typically as  $u_r \propto (E_\nu E_\eta / \Lambda)^{1/2} u_\phi$  when  $B_r \neq 0$  (i.e. away from the equator) and as  $u_r \propto (E_\nu E_\eta / \Lambda)^{1/4} u_\phi$  when  $B_r \rightarrow 0$  (i.e. near the equator). Here,  $\Lambda = v_A^2 / r_\odot^2 \Omega_{eq}^2$  where  $v_A$  is the Alfvén velocity. It is important to note that the gradual reduction in the magnetic diffusivity does not necessarily result in a concurrent increase in the magnetic Reynolds number: here, I find that as  $f \rightarrow 1$  the meridional flow velocities rapidly decrease and fail to confine the field. The radiative zone approaches a differentially rotating state constrained by Ferraro isorotation with the field, contrary to what is observed in the Sun.

It is possible to investigate instead situations where flows are forced from the convection zone down into the radiative zone as first suggested by Gough & McIntyre (1998) and later by Kitchatinov & Ruediger (2006).

Gough & McIntyre consider flows with a significant radial component while the model of Kitchatinov & Ruediger involves the latitudinal component of the flow only. In this preliminary study, I select a combination of radial and latitudinal forcing by setting<sup>1</sup>  $u_r(\theta) = 10^2(1 - 3\cos^2\theta)\text{cm/s}$

<sup>1</sup> The latitudinal component of this flow is equatorward, with an amplitude consistent with what one may expect of meridional flows within

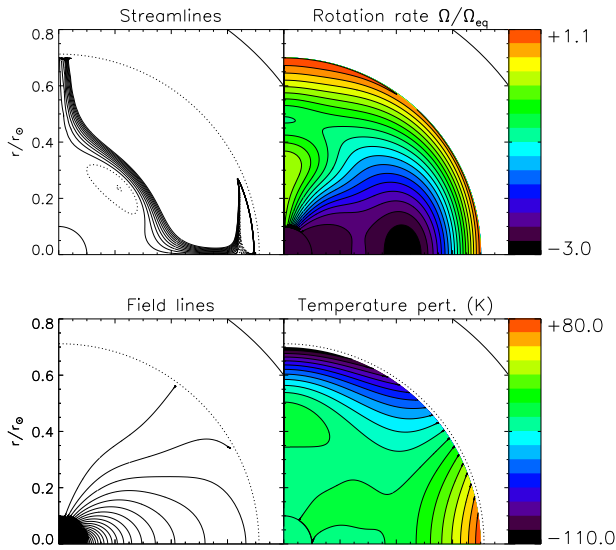
and  $u_\theta(\theta) = 10^3 \sin\theta \cos\theta \text{cm/s}$  at  $r = 0.7r_\odot$ . It is interesting to note that for a wide range of boundary conditions tested, a strong secondary circulation cell appears near the equator so that the global structure of the flow *within* the radiative zone is a two-cell circulation downwelling near the pole and the equator, with upwelling in mid-latitudes (see Fig. 2). The ubiquity of this somewhat surprising result is also seen in the work of Rieutord (2006) and Garaud & Brummell (2007) in the case of purely hydrodynamic flows, and remains to be fully explained. Thus in practise, one should view the flows somewhat below the outer boundary to downwell near both pole and equator, and upwell in mid-latitudes as predicted by Gough & McIntyre.

When  $f$  is gradually reduced below  $10^{10}$  the magnetic Reynolds number  $Rm = |u_r(0)|r_\odot/f\eta$  at the outer boundary begins to exceed unity, and nonlinear interactions between the field and the large-scale flows can dominate the tachocline dynamics. A gradual confinement of the field is observed this time, notably more pronounced near the equator (where the field lines are already inclined with respect to the vertical direction) than near the pole (where the field lines remain mostly radial). However, one can also clearly see that the angular velocity of the interior in this case is dramatically lower than that of the surface, which is a consequence of the advection of negative angular-momentum from the polar regions to lower latitudes by the meridional flows. The failure of this particular model setup to reproduce helioseismic inversions can be used to infer that a combination of the following two statements must be true: (i) the flow pattern at the radiative-convective interface differs widely from the one assumed here and (ii) turbulent stresses within the radiative zone are not negligible.

## 5 A quantitative look at models of the tachocline

Helioseismic inversions provide stringent constraints on angular-momentum transport in the solar interior, and in particular in the tachocline. To zeroth order, any model should be able to explain the near-uniform rotation of the solar radiative zone. To first order, a good model must be able to explain the value of the rotation rate of the interior (see equation (2)). To this date there have been only two published attempts at addressing the latter (Spiegel & Zahn 1992 and Garaud 2002), and in both cases the model predictions differ significantly from the observed value. The difficulty of the task lies, as mentioned earlier, partly in characterization of the unknown turbulent stresses, but also in the fact that angular-momentum advection by large-scale flows and the long-range nature of magnetic forces both imply that MHD models of the tachocline *cannot* be local; the dynamical structure of the radiative interior must be studied as a

the convection zone (Giles *et al.* 1997); the *imposed* radial component of the flow downwells near the poles and upwells near the equator, with an amplitude selected arbitrarily.



**Fig. 2** Same as Fig. 1 for  $f = 2 \times 10^9$  and for the same internal field strength, with an imposed large-scale meridional flow at the radiative-convective interface (see main text). The counter-rotation seen in the radiative zone is an unphysical consequence of the linearization of the inertial terms in the momentum equation and of the advection terms in the thermal energy equation. However, the strong deceleration of the deep equatorial regions remains a ubiquitous feature of this model setup when the whole set of nonlinear equations is solved.

whole<sup>2</sup>. The numerical tool that I have developed provides a mean to do this and to test quantitatively various MHD models of the tachocline against observations (e.g. of the angular velocity profile and of chemical mixing within the radiative zone). The results presented in §4 reveal some deficiencies in existing models.

Recently, T. Rogers and I have revisited the issue of the confinement of the primordial magnetic field through a different route. It has been known for many decades that the interaction between large-scale fields and small-scale turbulent fluid motions is far from trivial. An extensive study of the nonlinear transport of magnetic fields near a radiative-convective interface has been performed by Tobias *et al.* (2001) in Cartesian box simulations; their most striking conclusion is that regardless of the initial field configuration, turbulent transport always acts in such a way as to relax the system towards the same mean quasi-steady profile in which the amplitude of the magnetic flux is much reduced in regions of high turbulent kinetic energy. In other words, if the field is initially located mostly in the convective region, then it is actively pumped out of it, while if the field is initially located mostly in the radiative region, then it remains confined there by the turbulent motions. Following this idea T. Rogers performed the first simulations of the same process

<sup>2</sup> In fact, it would be preferable to study the combined dynamics of both radiative and convective regions in one single system.

in what can be thought of as a complete meridional slice of the Sun (including the radiative zone and most of the solar convective zone). Her results show a very clear confinement of the primordial field below the overshoot layer, with a drop in the magnetic energy in the large-scale component of the field by more than four orders of magnitude between the quiescent radiative regions just below  $0.69 r_{\odot}$  and the fully turbulent regions around  $0.73 r_{\odot}$ . To include this effect in my numerical simulations I have then replaced the usual field boundary conditions with  $B_r$  and  $B_{\phi}$  set to zero at the outer boundary of the numerical domain. The resulting agreement between the model predictions in terms of the interior angular velocity profile are in this case quite remarkable. These findings, which were also presented in the Potsdam workshop, are reported elsewhere in more detail (Garaud & Rogers, 2007).

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