A Statistical Equivalent Model for Random Waypoint Mobility: A Case Study

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Abstract

Statistical Equivalent Models, or SEMs, have recently attracted considerable interest as a general approach to study computer simulators. By fitting a statistical model to the simulator's output, SEMs provide an efficient way to quickly explore the simulator's result. In this paper, we develop a SEM for random waypoint mobility, one of the most widely used mobility models employed by network simulators in the evaluation of communication protocols for wireless multi-hop ad hoc networks (MANETs). We chose the random waypoint mobility model as a case study of SEMs due to recent results pointing out some serious drawbacks of the model (e.g., [1]). In particular, these studies show that, under the random waypoint mobility regime, average node speed tends to zero in steady state. They also show that average node speed varies considerably from the expected average value for the time scales under consideration in most simulation analysis.

In order to investigate further the behavior of the random waypoint model, we develop a SEM that captures speed decay over time under random waypoint mobility using maximum speed and terrain size as input parameters. A Bayesian approach to model fitting is employed to capture the uncertainty due to unknown parameters of the statistical model. The SEM is given by the posterior predictive distributions of the average node speed as a function of time. A direct result from our model is that, by characterizing average node speed as a function of time, our approach provides an accurate estimate of the "warm-up" period required by simulations using the random waypoint mobility model. Simulation data from the "warm-up" period can then be discarded to obtain accurate protocol performance results. Given that random waypoint mobility is still, by far, the most widely used mobility model in the evaluation of MANETs, the contribution of this work is potentially significant as it allows network protocol designers to continue to use the original random waypoint mobility model and yet obtain accurate results characterizing MANET protocol performance.

Keywords: Random Waypoint, Non-Linear Regression, Ad-Hoc Networks, Mobility Model

1 Introduction

The problem of fitting a statistical model to computer simulator results has recently been receiving considerable attention in the literature as an efficient way to perform fast exploration of the simulator's output. The method, referred to as Statistical Equivalent Modeling, consists of creating a relatively simple statistical model to approximate some function of the output of a computer simulator. The resulting Statistical Equivalent Model (SEM) must be flexible enough to capture most of the variability in the simulator's

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output. Since SEMs are used as surrogate models, it is important that the uncertainty in the SEM predictions be easily assessed in order to quantify its performance. SEMs are used for a variety of purposes which can be roughly classified in four groups: (1) model calibration to find ranges of the simulator input that produce sensible output; (2) data assimilation, which consists of merging simulator output with observations; (3) model validation for comparison of results from a simulator to observations or other simulators; and (4) model description and comparison for describing and comparing certain configurations of the simulator. The latter is the application of Statistical Equivalent Modeling we employ in the work described herein.

Regarding the literature on SEMs, the proposed methods, typically, involve Gaussian process response-surface approximations, i.e., the use of Gaussian processes to approximate the function that represents the computer simulator output (e.g., [2]). The literature includes a number of Bayesian approaches with a range of corresponding applications (see, e.g., [3, 4, 5, 6, 7, 8, 9]). Bayesian methodology is particularly suited to addressing the four issues discussed above, to quantifying multiple sources of error and uncertainty in computer simulators, and to combining multiple sources of information. We note that there has been relatively limited work on statistical methodology for the analysis of computer experiments under the practically important setting where the computer simulator produces functional output (see [10, 11] for some recent work in this direction).

The SEM we develop in this paper handles functional computer simulator data, in particular, data on the average node speed, as a function of time, based on the random waypoint mobility regime. This is a mobility model that has been extensively used by packet-level network simulators to study the performance of communication protocols for multi-hop wireless ad hoc networks (MANETs).

Packet-level network simulators (e.g., ns-2 [12], GloMoSim [13], QualNet [14], OPNET [15]) have been an extremely popular platform for evaluating MANET protocols. There are clear advantages to using simulations when evaluating network (in particular, MANET) protocols, including the ability to reproduce experiments and subject protocols to a wide range of network topologies and conditions, for example mobility patterns. Topology, number of network nodes and node mobility are important parameters that can significantly affect protocol performance.

Most existing network simulators employ random waypoint mobility to model how nodes move on a terrain [16]. Nodes in the random waypoint regime move according to the following rules: (1) each node picks a destination randomly within the simulation area and also picks a speed v that is uniformly chosen between v_{min} and v_{max}. Each node then moves toward the destination over a straight line with speed v. (2) upon reaching the destination, a node pauses for some *pause-time*; (3) the node then picks the next destination and the process re-starts. Typically, the values of v_{min}, v_{max}, and *pause-time* are parameters of the simulation and are selected according to the requirements and operating environment of the application at hand.

Recently, it has been reported that the random waypoint model exhibits some originally unforeseen anomalous behavior. More specifically, it has been shown that, under the random waypoint model, the average node speed decays with time [1]. It has also been shown that nodes moving according to the random waypoint model tend to concentrate in the middle of the simulation region, resulting in non-uniform node spatial distribution. In the specific case where $v_{min} = 0$, as time $t \to \infty$, node speeds tend to zero, resulting in a stationary system. One important effect of this behavior is that, if simulations using the random waypoint model do not run for sufficiently long periods beyond the initial steep decay, the corresponding simulation results will not be accurate. In fact, variations of up to 40% in ad hoc routing performance over a 900-second simulation have been detected [1].

From the above discussion, one important consideration is how long does it take for the system to converge to steady state. Given this information, one easy "fix" to the random waypoint model is to run simulations long enough to guarantee that protocol performance evaluation is conducted after steady state is reached to guarantee accurate results. In this paper, we introduce a novel approach to study the behavior of the random waypoint regime. We develop a SEM to predict average node speed (through both point and interval estimates) as a function of input parameters v_{max} and field size. Since our SEM also characterizes average node speed as a function of time, it offers an efficient alternative to obtaining an accurate estimate of how long simulation experiments take to "warm-up". Simulation data from the "warm-up" period can then be discarded to obtain accurate protocol performance results. Since random waypoint mobility continues to be, by far, the most widely used mobility model in the evaluation of MANETs, our model allows protocol designers to use the original random waypoint mobility model and still characterize MANET protocol performance accurately.

To build a SEM for random waypoint mobility, we consider a random waypoint simulator where the inputs are terrain size and maximum node speed. We run the simulator for a number of different configurations of these two variables. We then fit a statistical model to the resulting average node speed at different times. We validate the statistical model by comparing its predictions against the actual simulator results. We use the resulting model to measure the decay of average node speed and quantify the time that it takes for the "warm-up". We also show that our random waypoint mobility SEM is able to provide information on the warm-up period for different combinations of input parameters significantly faster than running pre-simulations of the mobility model for different input combinations. For instance, using our model, it took us 20 minutes to compute the point estimates of the warm-up period over a grid of values for v_{max} and field size (and for two different values of speed decay). Using the same (reasonably fast) machine, it would take approximately 65 hours to run pre-simulations for the same grid of v_{max} and field size values.

In the rest of the paper we present our statistical model in detail. Section 2 puts our work in perspective by describing related efforts in modeling the random waypoint regime. In Section 3, we present the methodology employed to formulate and develop the model. Section 4 describes the proposed statistical model. In Section 5, we present results obtained from the model and evaluate its accuracy by validating it against data obtained from the simulator. Finally, Section 6 presents concluding remarks as well as directions for future work.

2 Background and Related Work

Mobility models are an important component of network simulators and are one of the key factors affecting the performance of ad-hoc network protocols. A number of mobility models for ad-hoc networks have been proposed and evaluated ([17, 18, 19, 20]). One of the most widely used mobility models is the random waypoint model ([16, 21, 22]) described in Section 1. This model is implemented in a number of current network simulation platforms such as `ns-2` [12], `GloMoSim` [13], and `Qualnet` [14].

However, it has been shown in [1] that under the random waypoint regime, the average node speed decays with time before reaching steady state and the settling time to reach steady state increases as the minimum speed parameter v_{min} of the model decreases. In particular, the default random waypoint models distributed with `ns-2` and `GlomoSim` use $v_{min} = 0$ which causes the average node speed to steadily decrease over time. In [1], the impact of this speed decay on ad-hoc routing protocols like DSR [16] and AODV [23] was also investigated. It was shown that speed decay can result in performance variations of around 40% over simulation times typically used in the study of ad-hoc network protocols. One suggested solution was to use non-zero minimum speed or to discard results from the "burn-in" period, i.e., the simulation period

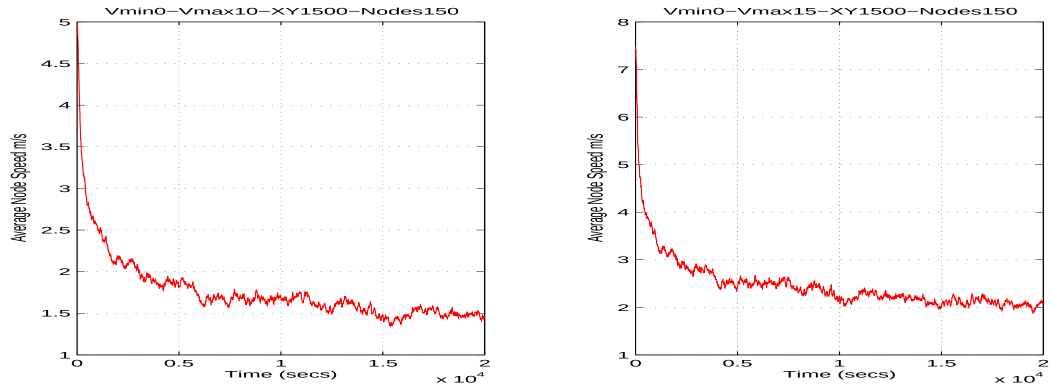


Figure 1: Speed decay under the random waypoint model

during which speed decay is most dramatic.

There have been several other bodies of work such as [24, 25, 26] which have investigated the spatial node distribution for the random waypoint model.

In [27], a framework for analyzing the speed decay of mobility models was proposed; additionally, based on this framework, a technique to obtain the stationary equivalent to mobility models that exhibit the speed decay behavior was introduced. Essentially, the proposed strategy is to choose initial speeds from the stationary distribution and subsequent speeds according to the original distribution. Similarly, in [28] the authors have used palm calculus to provide necessary and sufficient conditions for a stationary regime to exist under the random waypoint model and presented an algorithm to start simulations in the steady state (so called perfect simulation).

The main difference of our work compared to other approaches is that we propose a novel method to study the behavior of the random waypoint model which uses a statistical model to characterize speed decay. Our model is able to predict average node speed (through both point and interval estimates) as a function of input parameters v_{max} and field size. Our model also offers an efficient alternative to obtaining accurate results from simulations using the original Random Waypoint model [1]. More specifically, as it will become clear in Section 5.2, using our statistical model, one can obtain the speed decay as a function of time (as well as the input parameters). This allows protocol designers running simulations to plan their experiments accordingly so as to discard results from the "warm-up" period and hence perform accurate protocol performance evaluation.

[1]Note that the alternative is to run pre-simulations of the mobility model for different combinations of parameters of interest.

3 Methodology

We used `GloMoSim` [13] as the simulation platform for the initial mobility experiments. The simulation setup consisted of 150 nodes moving according to the random waypoint model with v_{max} from the set { 2, 3, 4, 5, 7.5, 10, 12.5, 15, 17.5 , 20 } m/s and $v_{min} = 0$. The pause-time was set to 0 for all experiments. The field-size was varied in the range { 500, 1000, 1500, 2000, 2500, 3000 } $m2$. Hence we ran mobility simulations for 60 different combinations of v_{max} and field-size with each run averaged over 10 different seed values. The total duration of the mobility experiments was set to 20,000 secs and we captured the average node speed as reported by the simulator every 5 secs. As noted previously the values were averaged over 10 different runs using different seed values. The data obtained from these mobility experiments was used as "computer simulator data" for our statistical model.

One key observation that helped to simplify model formulation was the implicit relationship between terrain size and number of nodes used in simulation experiments. For a given transmission range, the terrain size chosen normally dictates the minimum number of nodes required to ensure that the network is connected [2]. Hence our model implicitly accounts for number of nodes through the "field-size" parameter, which is defined as the 2-dimensional region within which nodes can move.

Figure 1 is a pictorial representation of the speed decay suffered by nodes using the Random Waypoint mobility model. Note that, the average initial speed of the nodes is $(v_{max} - v_{min})/2$ as expected and then starts decaying with time. This is similar to the results observed in [1].

4 Statistical Model

We fitted a statistical model to observations on average node speed, obtained from the simulator as discussed in section 3, for 10 different choices of v_{max} and 6 different field sizes. In what follows we use the notation v for v_{max}, f for field-size, t for time in seconds, and y_t for the average node speed at time t. We started by considering the non-linear regression model

$$y_t = \frac{c}{(1 + b(t/1000))^a} + \varepsilon, \tag{4.1}$$

where ε is a random error term, for each of the 60 combinations of v_{max} and field-size. We fitted these models using least squares and obtained a set of 60 triplets corresponding to the fitted values of a, b and c. By exploring the dependence of these values on v and f, we were able to generalize model (4.1) making the coefficients a, b and c dependent on v and f. Hence we obtain a SEM for the average node speed corresponding to any combination of v and f given by $y_t(v, f) = g(t, v, f; \mathbf{a}, \mathbf{b}, \mathbf{c}) + \varepsilon$, where

$$g(t, v, f; \mathbf{a}, \mathbf{b}, \mathbf{c}) = \frac{c(v, f)}{(1 + b(v, f)(t/1000))^{a(v,f)}} \tag{4.2}$$

with

$$\begin{aligned} a(v, f) &= \exp\{a_1 + a_2 \log(f/v) + a_3 \log(\log(f/v)) \\ &+ a_4 \log(\log((v/f) + 1)) \\ &+ a_5 \log(\log(v + 0.5))\} \\ b(v, f) &= \exp\{b_1 + b_2 \log v + b_3 \log f \\ &+ b_4 \log(\log(f/v)) + b_5 \log(\log f) \\ &+ b_6 \log(\log(v + 0.5))\} \\ c(v, f) &= \exp\{c_1 + c_2 \log v + c_3 \log f\} \end{aligned}$$

Here $\mathbf{a} = (a_1, \ldots, a_5)$, $\mathbf{b} = (b_1, \ldots, b_6)$ and $\mathbf{c} = (c_1, c_2, c_3)$ denote the vectors of the unknown coefficients. These can be estimated from the computer simulator data. This seemingly complicated structure was obtained in a constructive way, exploring many different alternatives and preferring those that provided the best fit with the smallest number of parameters. A critical advantage of this formulation is that, once the 14 unknown parameters are estimated, one can estimate the average node speed for any combination of field-size and v_{max}, and for any time.

The estimation of the parameters in the SEM was performed by assuming that the error term follows a normal distribution with zero mean and variance σ^2. Therefore, given the data $Y = \{y_t(v_i, f_j); t = 1, \ldots, T; i = 1, \ldots, 10; j = 1, \ldots, 6\}$, we obtain the likelihood for the parameter vector, which is denoted by $\boldsymbol{\theta} = (\mathbf{a}, \mathbf{b}, \mathbf{c}, \sigma^2)$, as

$$\begin{aligned} L(\boldsymbol{\theta}|Y) &= \prod_{t,i,j} (2\pi\sigma^2)^{-1/2} \exp\{-\frac{1}{2\sigma^2}(y_t(v_i, f_j) \\ &- g(t, v_i, f_j; \mathbf{a}, \mathbf{b}, \mathbf{c}))^2\} \end{aligned}$$

We estimate $\boldsymbol{\theta}$ using a Bayesian approach. This is based on exploring the posterior distribution $p(\boldsymbol{\theta}|Y)$. We consider a non-informative prior $p(\mathbf{a}, \mathbf{b}, \mathbf{c}, \sigma^2) \propto 1/\sigma^2$. Thus $p(\boldsymbol{\theta}|Y) \propto 1/\sigma^2 L(\boldsymbol{\theta}|Y)$. Under squared error loss, the optimal estimator is given by the posterior expectation $E(\boldsymbol{\theta}|Y)$.

Given the difficulties involved in describing, integrating or maximizing $p(\boldsymbol{\theta}|Y)$, which is a 15-dimensional function, we resort to Markov chain Monte Carlo (MCMC) methods to obtain samples from $p(\boldsymbol{\theta}|Y)$. The idea of MCMC methodology is to construct a

[2] Node mobility can cause the network to be disconnected at certain times

Markov chain that is easy to sample from and such that its equilibrium distribution is $p(\boldsymbol{\theta}|Y)$ [29]. Before we describe the Markov chain that we used, we note that $p(\mathbf{a}, \mathbf{b}, \mathbf{c}, \sigma^2|Y) = p(\sigma^2|\mathbf{a}, \mathbf{b}, \mathbf{c}, Y)p(\mathbf{a}, \mathbf{b}, \mathbf{c}|Y)$, where

$$\begin{aligned} p(\mathbf{a}, \mathbf{b}, \mathbf{c}|Y) &\propto A^{-60T/2} \\ p(\sigma^2|\mathbf{a}, \mathbf{b}, \mathbf{c}, Y) &\propto (\sigma^2)^{-(60T+2)/2} \exp\{-A/(2\sigma^2)\}, \end{aligned}$$

with $A = \sum_{t,i,j}(y_t(v_i, f_j) - g(t, v_i, f_j; \mathbf{a}, \mathbf{b}, \mathbf{c}))^2$. Thus we recognize $p(\sigma^2|\mathbf{a}, \mathbf{b}, \mathbf{c}, Y)$ as the density of an inverse gamma distribution with shape $60T/2$ and scale $A/2$.

To obtain samples from the posterior $p(\boldsymbol{\theta}|y)$ we follow the steps:

1. Set initial values $\boldsymbol{\theta}_0$ and total number of iterations K

2. Loop for $k = 1, \ldots, K$

3. At iteration k, denote the current samples with the super-index k, and sample a vector of candidates $(\mathbf{a}^*, \mathbf{b}^*, \mathbf{c}^*)$ from a normal distribution with mean $(\mathbf{a}^k, \mathbf{b}^k, \mathbf{c}^k)$ and covariance matrix $\mathbf{V}$.

4. Calculate $\alpha = \min\{1, r\}$ where
$$r = \frac{p(\mathbf{a}^*, \mathbf{b}^*, \mathbf{c}^*|Y)}{p(\mathbf{a}^k, \mathbf{b}^k, \mathbf{c}^k|Y)}$$

5. Sample u from a uniform distribution on (0,1).

6. If $u < \alpha$ then sample $(\sigma^2)^*$ from an inverse gamma distribution with shape $60T/2$ and scale $A^*/2$, where A^* denotes the evaluation of A at the candidate values $(\mathbf{a}^*, \mathbf{b}^*, \mathbf{c}^*)$. Let $(\mathbf{a}^{k+1}, \mathbf{b}^{k+1}, \mathbf{c}^{k+1}) = (\mathbf{a}^*, \mathbf{b}^*, \mathbf{c}^*)$, $(\sigma^2)^{k+1} = (\sigma^2)^*$ and cycle.

7. If $u > \alpha$, let $(\mathbf{a}^{k+1}, \mathbf{b}^{k+1}, \mathbf{c}^{k+1}) = (\mathbf{a}^k, \mathbf{b}^k, \mathbf{c}^k)$, $(\sigma^2)^{k+1} = (\sigma^2)^k$ and cycle.

After an initial burn-in period, the results from this chain yield a sequence of samples $\boldsymbol{\theta}^k$ whose distribution is approximately $p(\boldsymbol{\theta}|Y)$. These posterior samples can be used to obtain inference for $\boldsymbol{\theta}$.

5 Results

In this section we present results obtained from the statistical model and assess its performance using mobility data obtained from the simulator. As explained in section 4, we ran a Markov Chain Monte Carlo (MCMC) algorithm in MATLAB to obtain samples from the posterior distribution for $p(\boldsymbol{\theta}|Y)$. These samples were then used to estimate $a(v, f), b(v, f)$ and $c(v, f)$ for different combinations of v_{max} and field-size. The estimates thus obtained were then used to evaluate for each combination of (v, f) the posterior mean of y_t as given in equation (4.2) for 4000 time-points up to 20,000 secs. Note that for each combination of (v, f) we obtain samples from the entire posterior distribution for equation (4.2). We present both point estimates and interval estimates (denoted by dashed lines in the subsequent figures) based on 5% and 95% quantiles of the posterior samples.

Figure 2 depicts the comparison between the simulator data and posterior point and interval estimates based on the statistical model for v_{max} = 2 m/s, while figures 3 and 4 show the comparison for v_{max} = 10 m/s and 20 m/s, respectively. Note that, in the figures we only present model fits up-to 15,000 secs for the sake of clarity as the behavior beyond 15,000 seconds is very similar.

As seen from these figures, the statistical model produces good fits as compared to the mobility data from the simulator. The interval estimates tend to capture the variability of the original data as well. One minor discrepancy is the tendency of the statistical model to overestimate the actual values of the average node speed at t close to 0.

5.1 Model Validation

In order to verify the accuracy of the proposed estimator we also ran some validation tests. In these tests we used the random waypoint mobility SEM of section 4 to estimate average node speed over time for values of v_{max} that are not included in the set of 10 values used in developing the SEM. We then validate the SEM predictions against computer simulator data obtained under

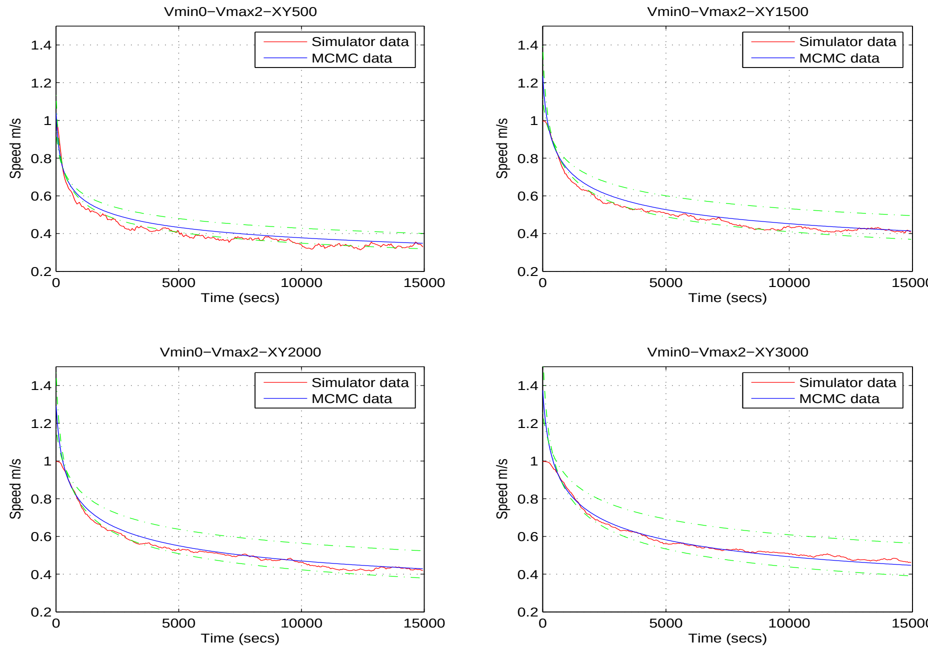


Figure 2: Inference: v_{max} 2 m/s

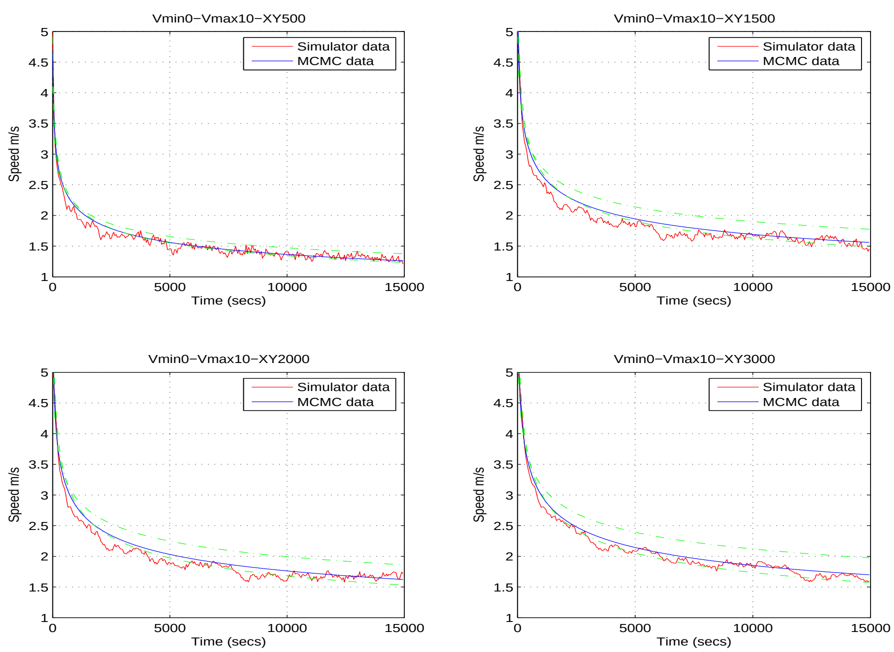


Figure 3: Inference: v_{max} 10 m/s

the new v_{max} values. In particular, we used two different values of v_{max}, i.e, {8, 25}, keeping all other simulator parameters constant. Note that one of the v_{max} values, i.e, 8 m/s is within the data range originally considered while the other value, i.e, 25 m/s is outside the data range used

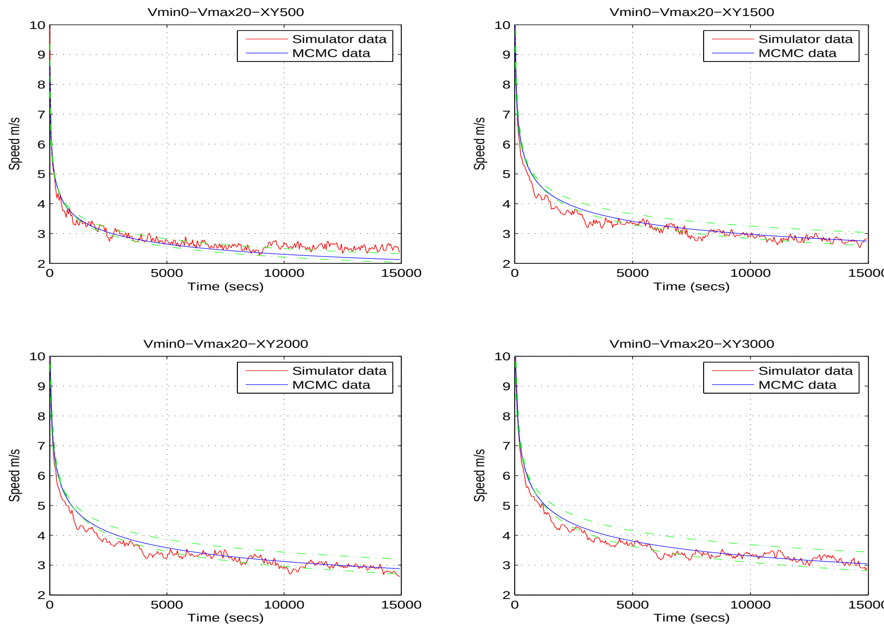


Figure 4: Inference: v_{max} 20 m/s

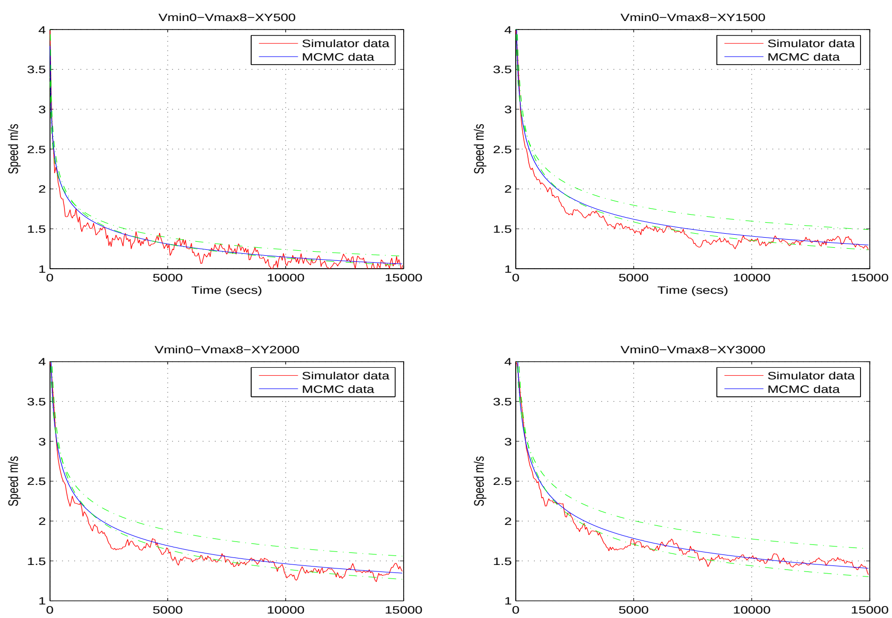


Figure 5: Validation v_{max} 8 m/s

to formulate the SEM. Figures 5 and 6 illustrate that the statistical model provides good fits for the new simulator data as well.

5.2 Discussion

As mentioned in Section 1, the main contribution of this work is the ability of the statistical model

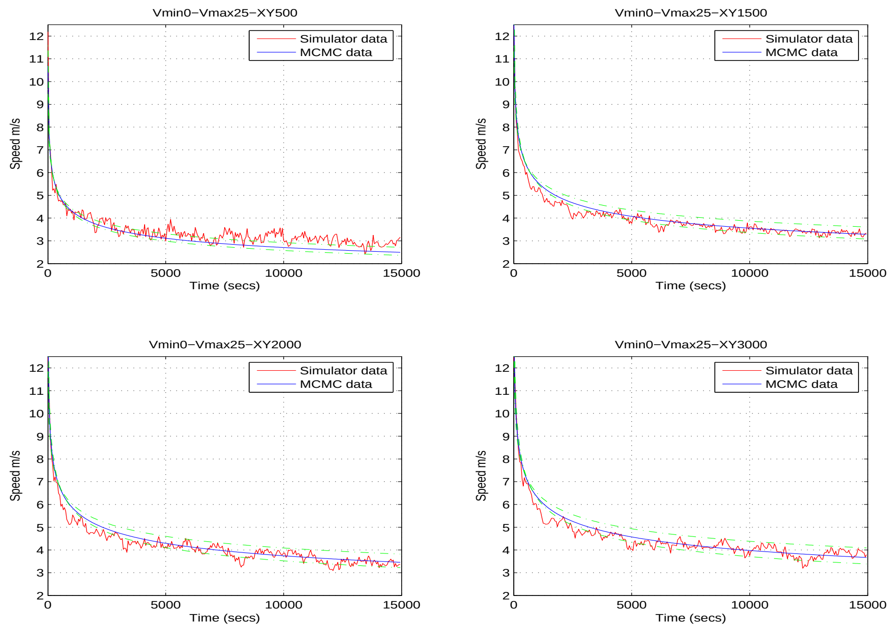


Figure 6: Validation v_{max} 25 m/s

to predict the average node speed (through both point and interval estimates) as a function of input parameters v_{max} and field-size. One of the recommended techniques to obtain accurate simulation results using the random waypoint model is to discard results from the "warm-up" period during which average node speed is still decaying. The proposed statistical model is useful in providing inference for the "warm-up" period for a specific simulation using the following equation

$$t_{warm-up} = 1000.b^{-1}\{(c/y_t)^{a^{-1}} - 1\},$$

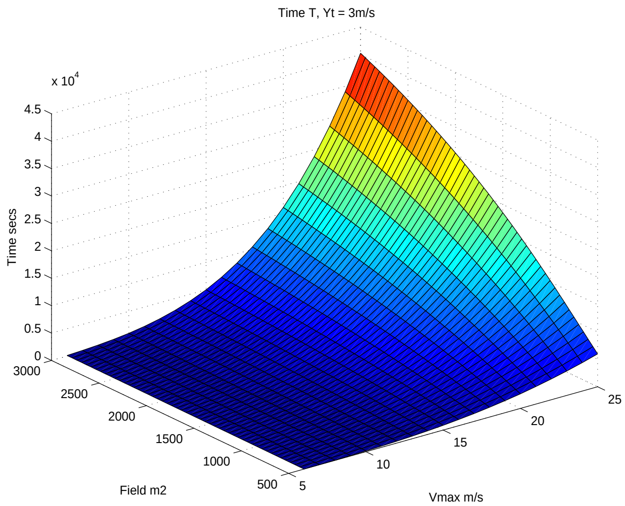
where y_t is the required value for the speed decay and a, b and c are functions of v_{max} and field-size as defined in section 4. Hence we can obtain the entire posterior distribution for $t_{warm-up}$ as a function of v_{max} and field-size for different values of speed decay y_t.

Figure 7 represents the point estimates of the "warm-up" period for a grid (of size 1250) over a range of commonly used combinations of v_{max} and field-size for 2 different values of y_t.

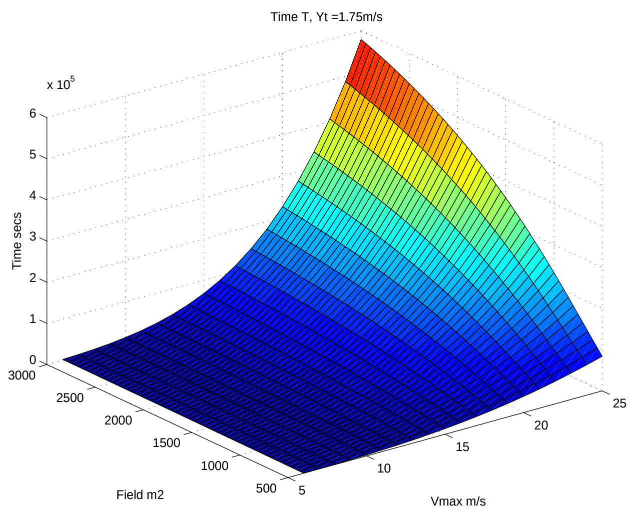
To put these results in perspective, the alternative approach would require running pre-simulations of the mobility model. For the 1250 different combinations of v_{max} and field-size considered above this would take approximately 65 hrs for 10 different seed values on a sufficiently fast simulation machine, whereas our approach required approximately 20 minutes of computing time.

6 Conclusions

This paper conducts a case study of Statistical Equivalent Modeling applied to the Random Waypoint Mobility model used in network simulators. This novel modeling technique of characterizing the behavior of the Random Waypoint Mobility regime captures speed decay over time using maximum speed and terrain size as input parameters. A Bayesian approach to model fitting is employed to capture the uncertainty due to unknown parameters of the statistical model. The resulting posterior predictive distributions of the quantities of interest (i.e, average node speed) can be used to formally address the fit of the statistical model. We present results obtained from the model and evaluate its accuracy by validating it against data obtained from the simulator.



(a) Warm-up times for speed decay of $Y_t = 3$ m/s



(b) Warm-up times for speed decay of $Y_t = 1.75$ m/s

Figure 7: Point Estimates of $t_{warm-up}$ as a function of v_{max} and field size

One of the main contributions of our random waypoint mobility SEM is that it offers an efficient alternative to circumventing recently uncovered anomalies of random waypoint mobility, one of the most widely used mobility models for evaluating the performance of multi-hop wireless ad hoc networks (MANETs). These anomalies include the fact that average node speed tends to zero as $t \rightarrow \infty$ and that node speed varies considerably from the expected average value for the time scales under consideration for most simulation analysis. Since our model characterizes average node speed as a function of time, it provides an accurate estimate of the time it takes for random waypoint mobility simulations to "warm up", i.e., reach steady state. Using this information, MANET protocol designers can continue to use the original random waypoint mobility model, discard simulation data produced during the "warm-up" period and still obtain accurate performance results for the protocols under study. This is an important contribution given that original Random Waypoint Mobility is still, by far, the most widely used mobility model in the evaluation of MANETs [30], [31], [32].

We also show that our random waypoint mobility SEM is able to provide information on the warm-up period for different combinations of input parameters significantly faster than running pre-simulations of the mobility model for different input combinations. For instance, using our model, it took us 20 minutes to compute the point estimates of the warm-up period as a function of v_{max} and field size for two different values of speed decay. Using the same (reasonably fast) machine, it would take close to 200 times longer to run pre-simulations for the same number of combinations of v_{max} and field size values.

One direction of future work involves extending the random waypoint mobility SEM to incorporate non-zero minimum speeds ($v_{min} \neq 0$).

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